CS 610: Write Cache-Friendly Code

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Content influenced by many excellent references, see References slide for acknowledgements.

Let us compare the performance!

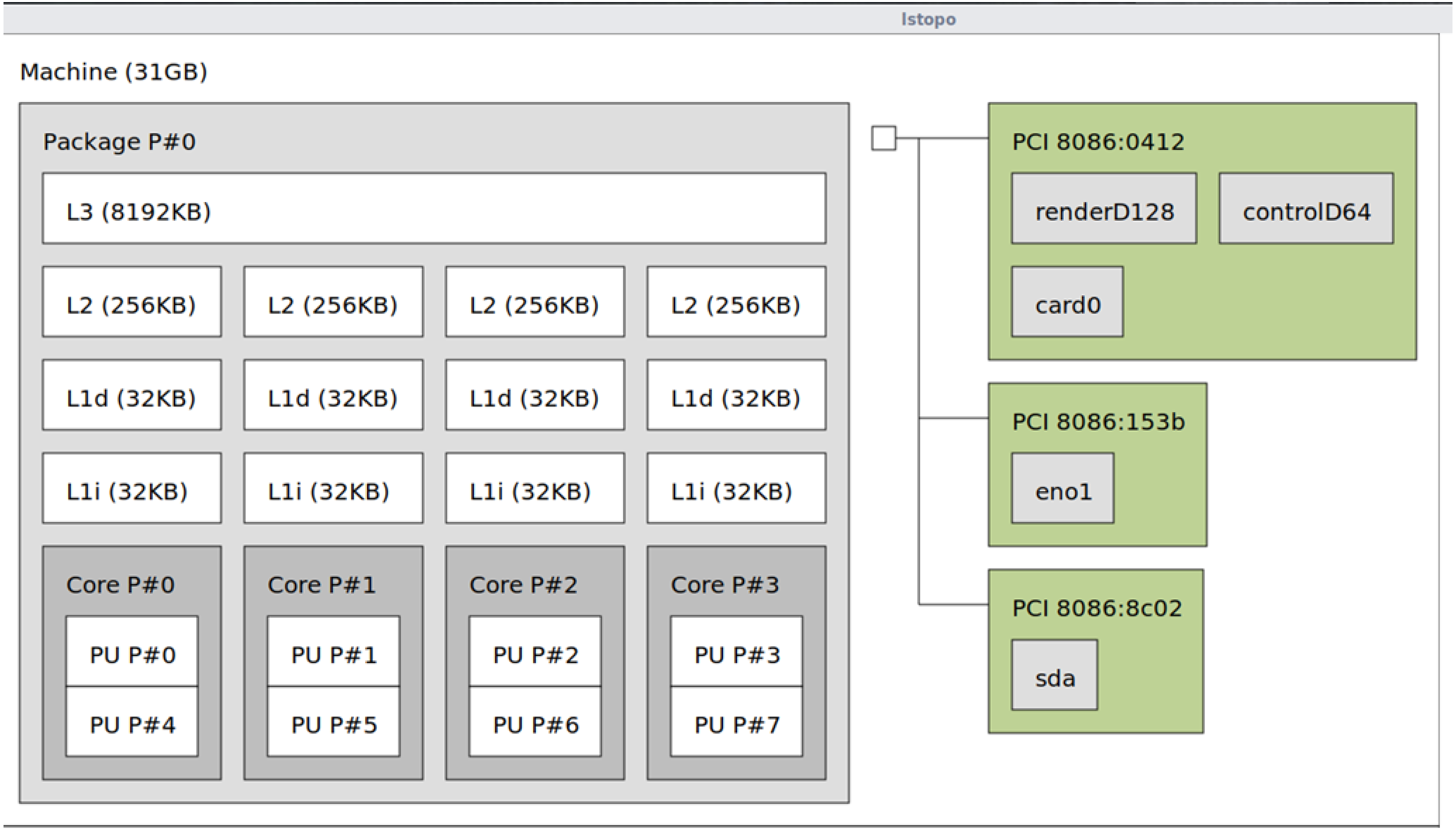
```
for (i = 0; i < 100000000; i++) {
  W = 1.599999 * X;
  X = 0.999999 * W;
}
```

550-600 ms

```
for (i = 0; i < 100000000; i++) {
  W = 1.599999 * W + 0.000001;
  X = 0.999999 * X;
  Y = 3.14159 * Y + 0.000001;
  Z = Z + 1.0001;
}
```

??? ms

Adapted from CS 5441 by P. Sadayappan @ Ohio State University


lstopo
Machine (31GB)
Package P#0
L3 (8192KB)
L2 (256KB)
L2 (256KB)
L2 (256KB)
L2 (256KB)
L1d (32KB)
L1d (32KB)
L1d (32KB)
L1d (32KB)
L1i (32KB)
L1i (32KB)
L1i (32KB)
L1i (32KB)
Core P#0
PU P#0
PU P#4
Core P#1
PU P#1
PU P#5
Core P#2
PU P#2
PU P#6
Core P#3
PU P#3
PU P#7
PCI 8086:0412
renderD128
controlD64
card0
PCI 8086:153b
eno1
PCI 8086:8c02
sda

Let us compare the performance!

```
#define N 32
#define T 1024 * 1024
double A[N][N];

for (it = 0; it < T; it++)
  for (j = 0; j < N; j++)
    for (i = 0; i < N; i++)
      A[i][j] += 1;
```

- `#define N 32`
- `#define T 1024 * 1024`

235 ms

- `#define N` **`128`**
- `#define T 1024 * 1024`

240 ms

- `#define N` **`256`**
- `#define T 1024 * 1024`

420 ms

- `#define N` **`4096`**
- `#define T 1024 * 1024`

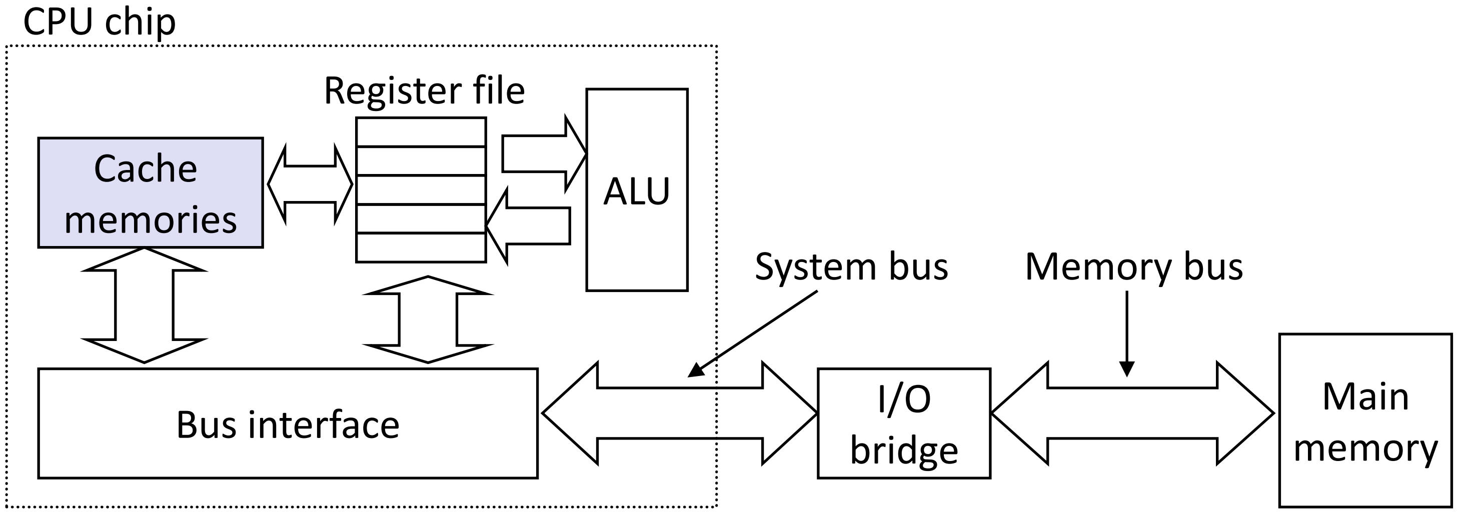
750 ms

Cache Memory: Quick Recap

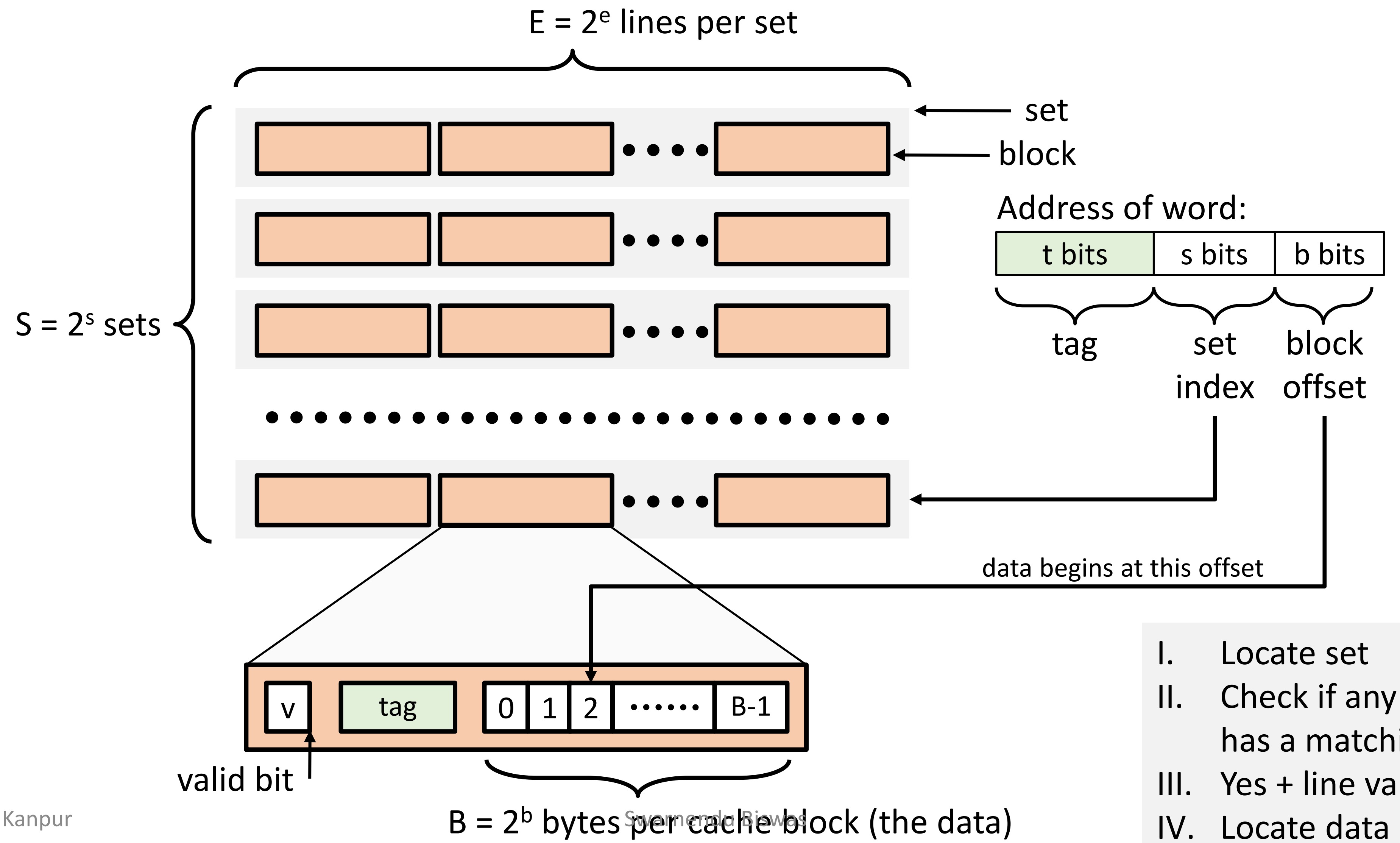
Slides adapted from Bryant and O'Hallaron (CS 15-213 @ CMU)

Cache Memories

- Cache memories are small, fast SRAM-based memories managed automatically in hardware and hold frequently accessed blocks of main memory
  - CPU looks first for data in caches (e.g., L1, L2, and L3), then in main memory
  - Because of locality, programs tend to access the data at level k (higher) more often than they access the data at level k+1



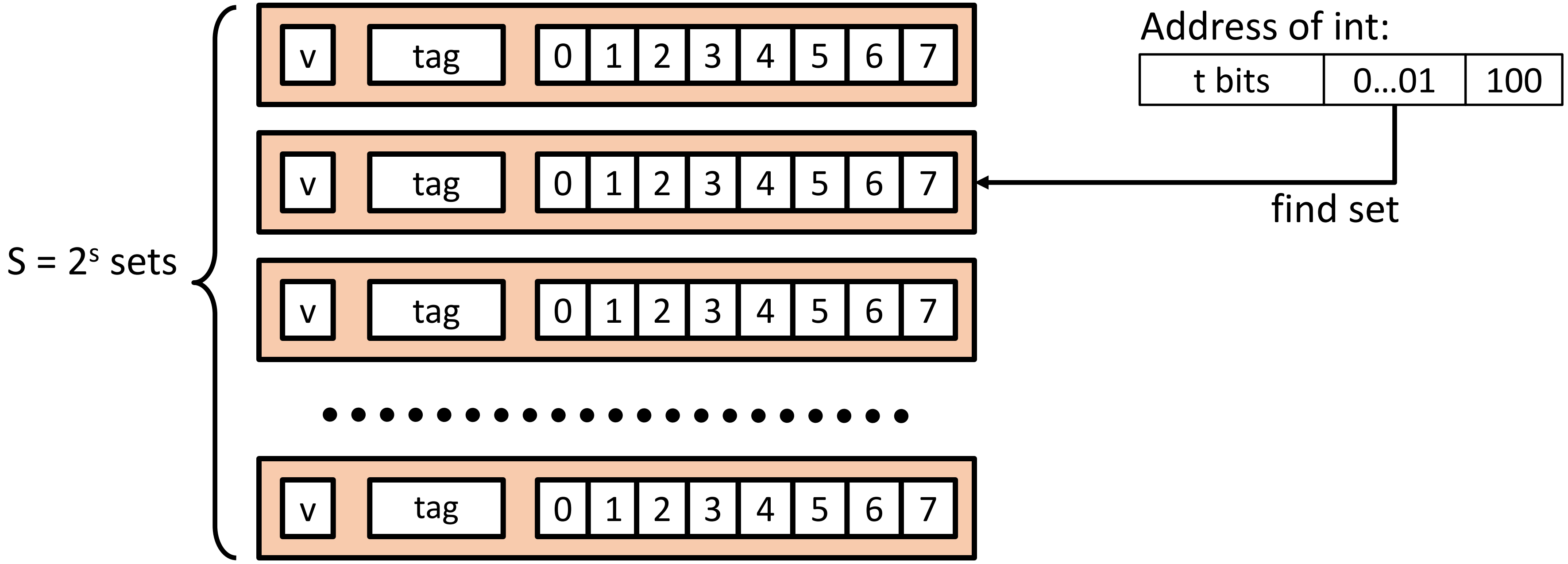
Cache Organization and Lookup



I. Locate set
II. Check if any line in the set has a matching tag
III. Yes + line valid: hit
IV. Locate data starting at offset

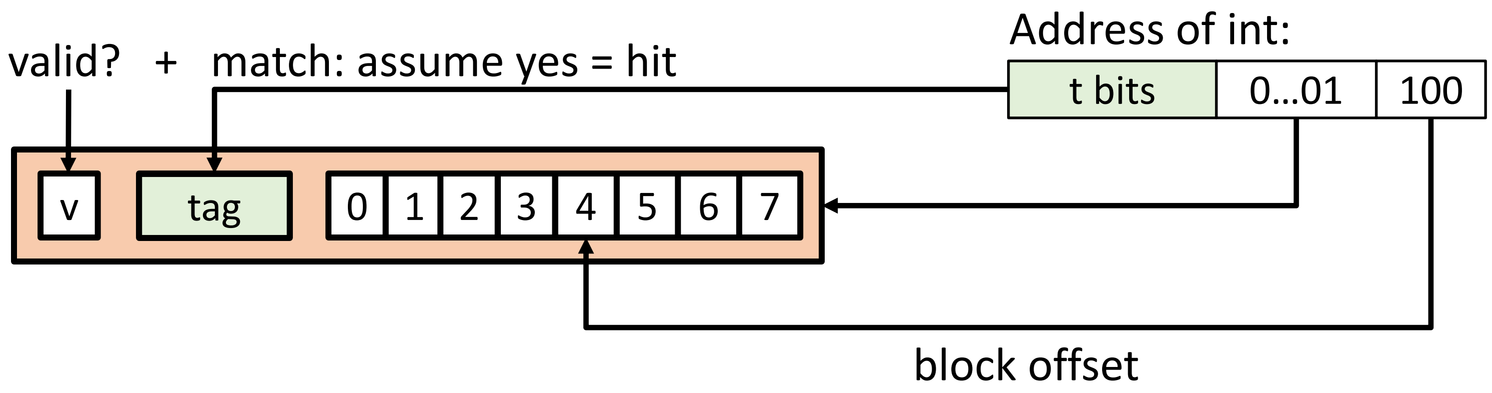
Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set
Assume: cache block size 8 bytes



Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set
Assume: cache block size 8 bytes



Direct-Mapped Cache Simulation

t=1	s=2	b=1
x	xx	x

M=16 byte addresses, B=2 bytes/block,
S=4 sets, E=1 Blocks/set

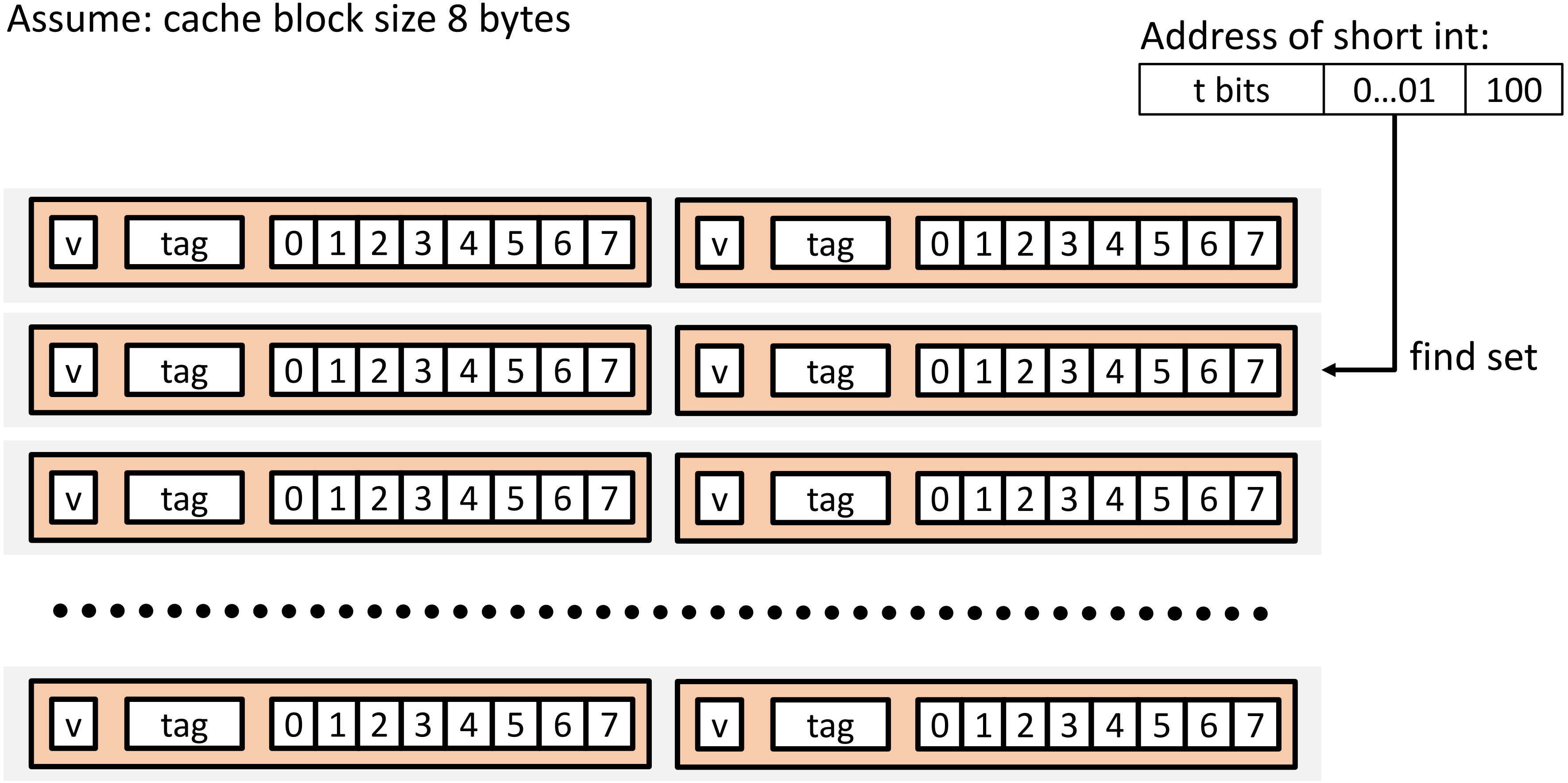
Address trace (reads, one byte per read):

0	$[0\underline{00}0_2]$,
1	$[0\underline{00}1_2]$,
7	$[0\underline{11}1_2]$,
8	$[1\underline{00}0_2]$,
0	$[0\underline{00}0_2]$

	v	Tag	Block
Set 0			
Set 1			
Set 2			
Set 3			

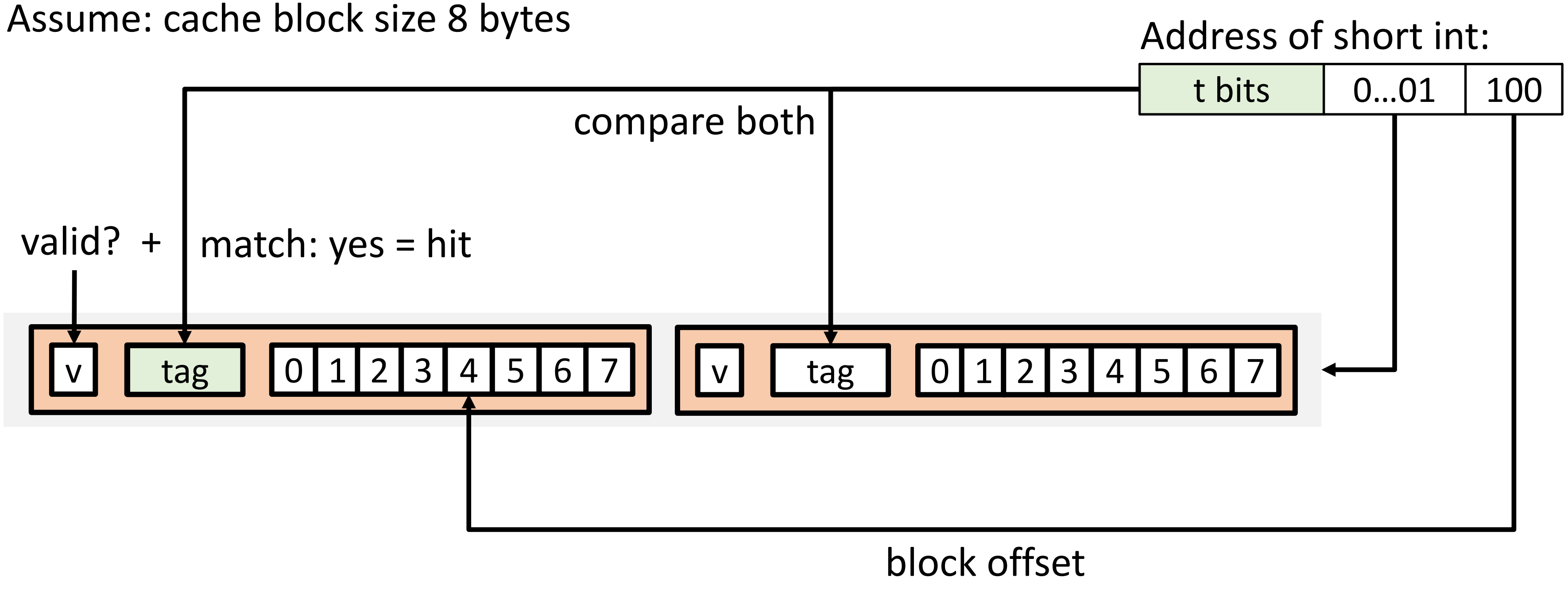
E-way Set Associative Cache (E = 2)

E = 2: Two lines per set
Assume: cache block size 8 bytes



E-way Set Associative Cache (E = 2)

E = 2: Two lines per set
Assume: cache block size 8 bytes



2-Way Set Associative Cache Simulation

t=2	s=1	b=1
xx	x	x

M=16 byte addresses, B=2 bytes/block,
S=2 sets, E=2 blocks/set

Address trace (reads, one byte per read):

0	$[00\underline{0}0_2]$,
1	$[00\underline{0}1_2]$,
7	$[01\underline{1}1_2]$,
8	$[10\underline{0}0_2]$,
0	$[00\underline{0}0_2]$

	v	Tag	Block
Set 0	0		
	0		
Set 1	0		
	0		

Evaluating Cache Performance

Miss rate

- Fraction of memory references not found in cache (misses/access)

Hit time

- Time to deliver a line in the cache to the processor, including the time to determine whether the line is in the cache

Miss penalty

- Additional time required because of a miss

Average Memory Access Time

- AMAT = $time_{hit}$ + $prob_{miss}$ * $penalty_{miss}$
- Let us compare performance of 99% and 97% hit rates
  - Consider cache hit time of 1 cycle
  - Miss penalty of 100 cycles
- $AMAT_{99\%}$ = ?
- $AMAT_{97\%}$ = ?

Average Memory Access Time

- AMAT = $time_{hit} + prob_{miss} * penalty_{miss}$
- Let us compare performance of 99% hit rate with 97%
  - Consider cache hit time of 1 cycle
  - Miss penalty of 100 cycles
- $AMAT_{99\%} = 1 + 0.01*100 = 2$ cycles
- $AMAT_{97\%} = 1 + 0.03*100 = 4$ cycles

- For multilevel cache
  - $AMAT_i$ (at level i) = $time_{hit_i} + prob_{miss_i} * AMAT_{i-1}$

Write Cache-Friendly Code

Slides adapted from Bryant and O’Hallaron (CS 15-213 @ CMU)

Is this function cache friendly?

```
int sumvec(int v[N]) {
  int sum=0;
  for (int i = 0; i < N; i++) {
    sum += v[i];
  }
  return sum;
}
```

Suppose v is block-aligned, words are 4 bytes, cache blocks are 4 words, and the cache is initially empty.

What can you say about locality of variables `i`, `sum`, and elements of v?

Is this function cache friendly?

```
int sumvec(int v[N]) {
  int sum=0;
  for (int i = 0; i < N; i++) {
    sum += v[i];
  }
  return sum;
}
```

ADDR	0	4	8	12	16	20
Contents	v_0	v_1	v_2	v_3	v_4	V_5
Iteration	0	1	2	3	4	5
Hit/miss	Miss	Hit	Hit	Hit	Miss	Hit

Compare the two programs

```
for (int i = 0; i < n; i++) {
  z[i] = x[i] – y[i];
  z[i] = z[i] * z[i];
}
```

```
for (int i = 0; i < n; i++) {
  z[i] = x[i] – y[i];
}
for (int i = 0; i < n; i++) {
  z[i] = z[i] * z[i];
}
```

Which version is more efficient if we have **large** arrays?

Data Locality

Parallelism and data locality go hand-in-hand

- Repeated references to memory locations or variables are good – temporal locality
- Stride-1 reference patterns are good – spatial locality

Always focus on optimizing the common case

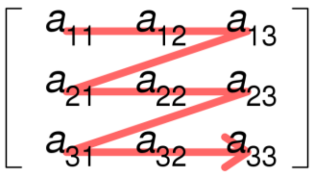
Layout of C Arrays in Memory

- C arrays are allocated in row-major order
  - Stepping through columns in one row exploits spatial locality if block size (B) > 4 bytes

```
int A[N][N];

for (i = 0; i < N; i++)
    sum += A[0][i];
```

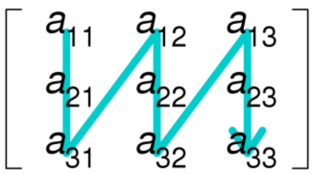
Row-major order



- Stepping through rows in one column
  - Accesses distant elements, no spatial locality!

```
for (i = 0; i < n; i++)
    sum += A[i][0];
```

Column-major order



Row- and column-major order

Compare Access Strides

Assume words are 4 bytes, cache blocks are 4 words, and the cache is initially empty.

```
int sumarrayrows(int a[M][N]) {
  int i, j, sum=0;
  for (i = 0; i < M; i++)
    for (j = 0; j < N; j++)
      sum += A[i][j];
  return sum;
}
```

```
int sumarraycols(int a[M][N]) {
  int i, j, sum=0;
  for (j = 0; j < M; j++)
    for (i = 0; i < N; i++)
      sum += A[i][j];
  return sum;
}
```

What are the miss rates per iteration if the array A (i) fits in cache and (ii) does not fit in cache?

Compare Access Strides

```
int sumarrayrows(int a[M][N]) {
  int i, j, sum=0;
  for (i = 0; i < M; i++)
    for (j = 0; j < N; j++)
      sum += A[i][j];
  return sum;
}
```

```
int sumarraycols(int a[M][N]) {
  int i, j, sum=0;
  for (j = 0; j < M; j++)
    for (i = 0; i < N; i++)
      sum += A[i][j];
  return sum;
}
```

4-20X
slower

Miss Rate Analysis for Matrix-Matrix Multiply

- Matrix-Vector multiply and Matrix-Matrix multiply are important kernels
  - Heavily used in ML and computational science applications
- Multiply NxN matrices with $O(N^3)$ operations
  - N reads per source element
  - N values summed per destination
  - `sum` can be stored in a register
  - $3N^2$ memory locations
- Algorithm is **computation-bound**
  - Memory accesses should not constitute a bottleneck

```
/* ijk */

for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++) {
      sum += A[i][k] * B[k][j];
    }
    C[i][j] = sum;
  }
}
```

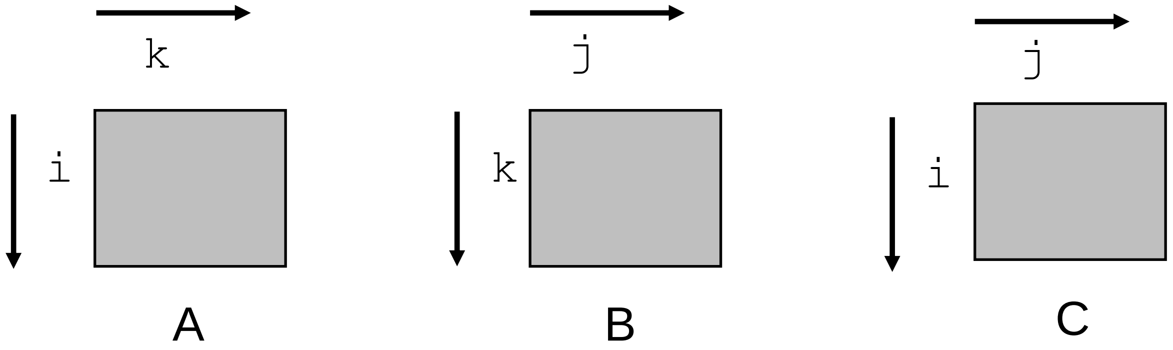
Cache Model

- Assumptions:
  - Only consider cold and capacity misses, ignore conflict misses
  - Large cache model: only cold misses
  - Small cache model: both cold and capacity misses

  - Line size = 32B (big enough for four 64-bit words)
  - Matrix dimension (i.e., N) is very large
    - Approximate $\frac{1}{N}$ as 0.0
  - Cache is not even big enough to hold multiple rows

Miss Rate Analysis for Matrix Multiply

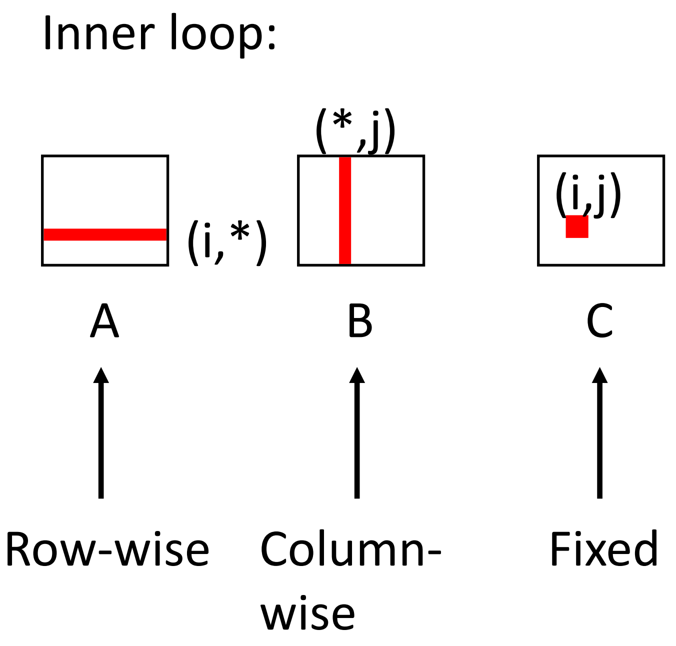
- Analysis Method:
  - Look at access patterns of the inner loop



Matrix Multiplication (ijk)

```
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += A[i][k] * B[k][j];
    C[i][j] = sum;
  }
}
```

two loads, zero stores

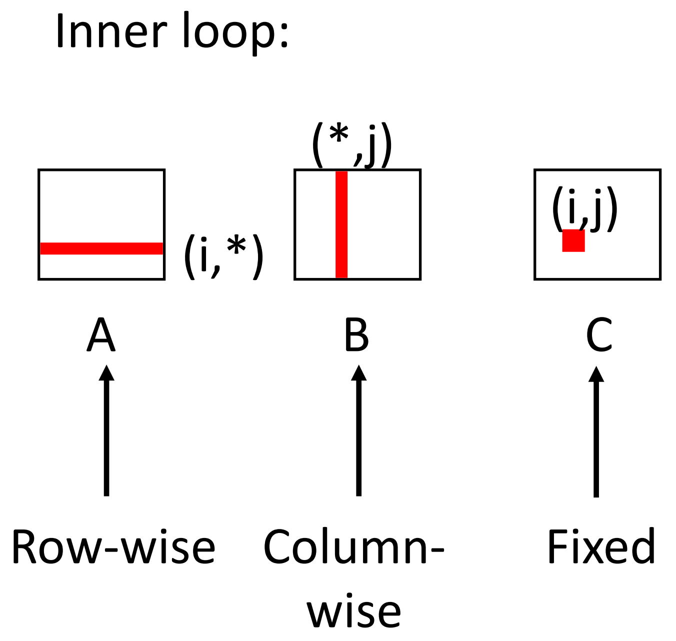


<u>Misses per inner loop iteration:</u>

<u>A</u>	<u>B</u>	<u>C</u>
0.25	1.0	0.0

Matrix Multiplication (jik)

```
for (j=0; j<n; j++) {
  for (i=0; i<n; i++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += A[i][k] * B[k][j];
    C[i][j] = sum
  }
}
```



Misses per inner loop iteration:

A	B	C
0.25	1.0	0.0

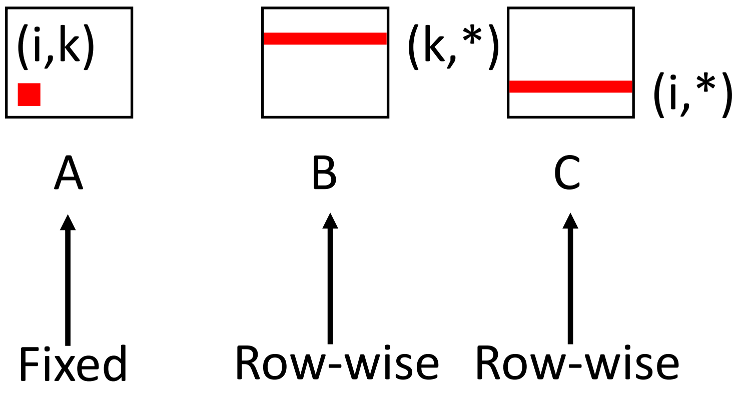
same behavior as ijk

Matrix Multiplication (kij)

two loads, one store

```
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = A[i][k];
    for (j=0; j<n; j++)
      C[i][j] += r * B[k][j];
  }
}
```

Inner loop:

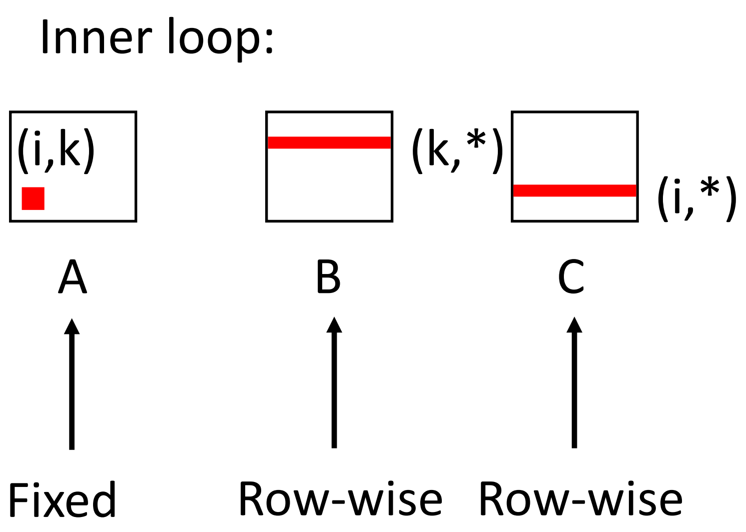


<u>Misses per inner loop iteration:</u>

<u>A</u>	<u>B</u>	<u>C</u>
0.0	0.25	0.25

Matrix Multiplication (ikj)

```
for (i=0; i<n; i++) {
  for (k=0; k<n; k++) {
    r = A[i][k];
    for (j=0; j<n; j++)
      C[i][j] += r * B[k][j];
  }
}
```



<u>Misses per inner loop iteration:</u>

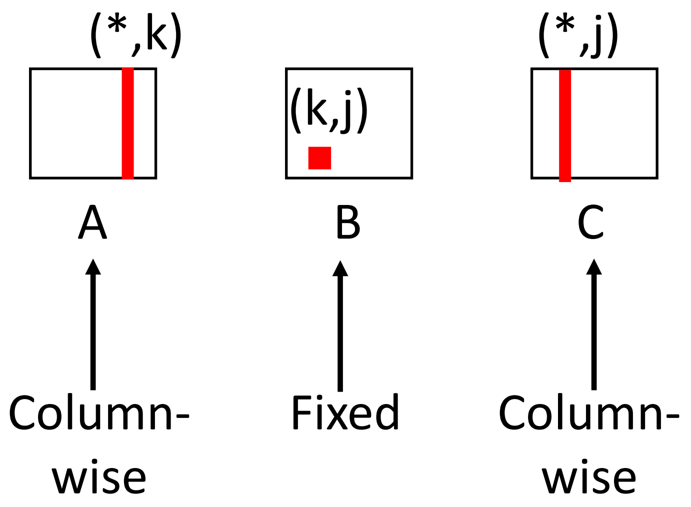
<u>A</u>	<u>B</u>	<u>C</u>
0.0	0.25	0.25

Matrix Multiplication (jki)

two loads, one store

```
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = B[k][j];
    for (i=0; i<n; i++)
      C[i][j] += A[i][k] * r;
  }
}
```

Inner loop:



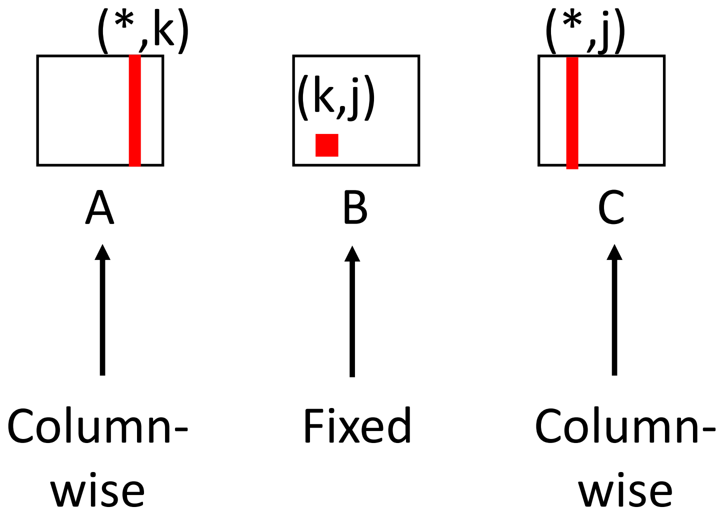
<u>Misses per inner loop iteration:</u>

A	B	C
1.0	0.0	1.0

Matrix Multiplication (kji)

```
for (k=0; k<n; k++) {
  for (j=0; j<n; j++) {
    r = B[k][j];
    for (i=0; i<n; i++)
      C[i][j] += A[i][k] * r;
  }
}
```

Inner loop:



<u>Misses per inner loop iteration:</u>

A	B	C
1.0	0.0	1.0

Summary of Misses Per Inner Loop Iteration

```
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
   sum = 0.0;
   for (k=0; k<n; k++)
     sum += A[i][k] * B[k][j];
   C[i][j] = sum;
 }
}
```

```
for (k=0; k<n; k++) {
 for (i=0; i<n; i++) {
  r = A[i][k];
  for (j=0; j<n; j++)
   C[i][j] += r * B[k][j];
 }
}
```

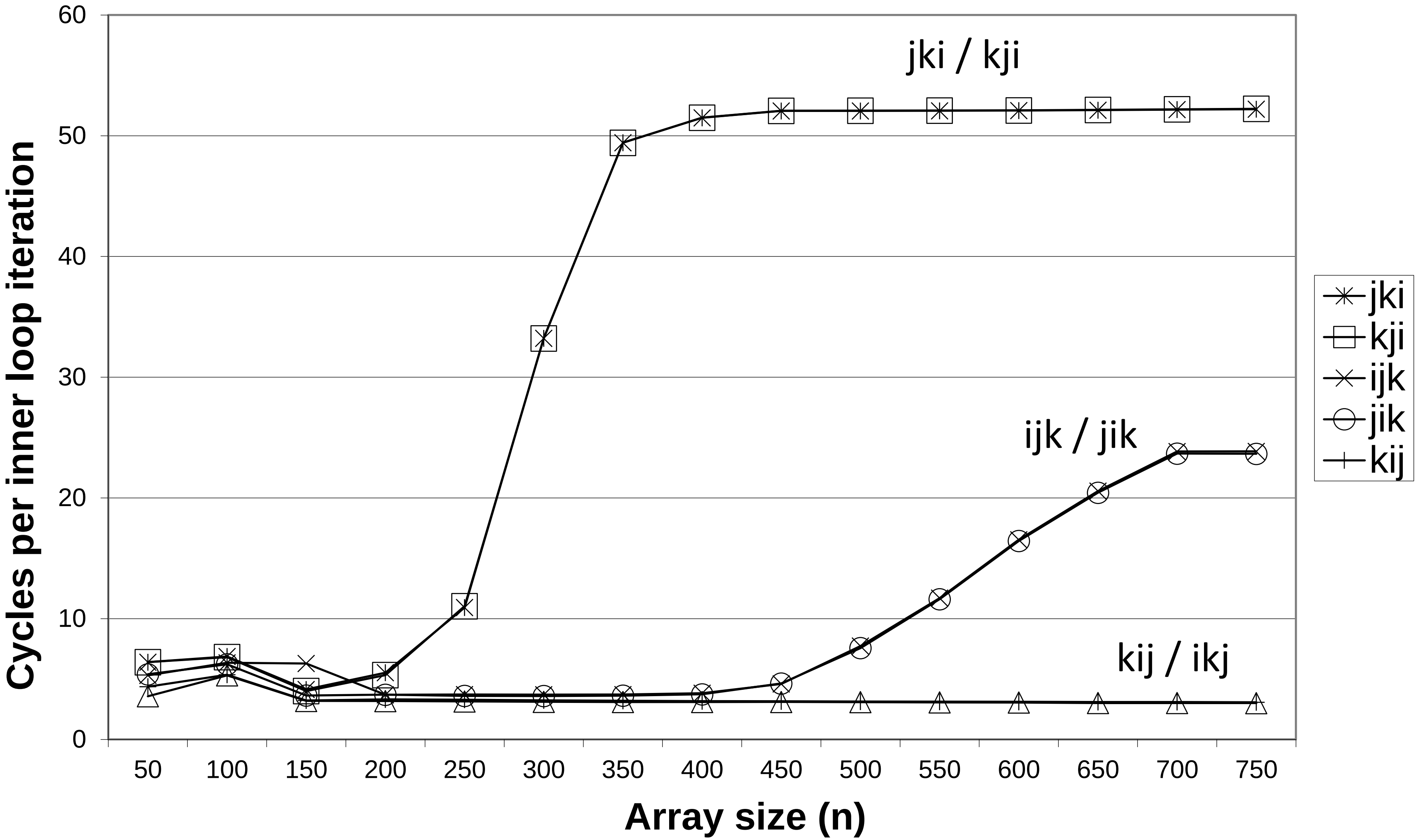
```
for (j=0; j<n; j++) {
 for (k=0; k<n; k++) {
   r = B[k][j];
   for (i=0; i<n; i++)
    C[i][j] += A[i][k] * r;
 }
}
```

`ijk (& jik):`
- 2 loads, 0 stores
- misses/iter = 1.25

`kij (& ikj):`
- 2 loads, 1 store
- misses/iter = 0.5

`jki (& kji):`
- 2 loads, 1 store
- misses/iter = 2.0

Matrix Multiply Performance on Core i7



Total Cache Misses (ijk)

Matrices are very large compared to cache size

```
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += A[i][k] * B[k][j];
    C[i][j] = sum;
  }
}
```

	A	**B**	**C**
I	?	?	?
J	?	?	?
K	?	?	?

Total Cache Misses (ijk)

Matrices are very large compared to cache size

```
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += A[i][k] * B[k][j];
    C[i][j] = sum;
  }
}
```

	A	**B**	**C**
I	n	n	n
J	n	n	n/BL
K	n/BL	n	1
	$\mathbf{n^3/BL}$	$\mathbf{n^3}$	$\mathbf{n^2/BL}$

Total Cache Misses (jki)

Matrices are very large compared to cache size

```
for (j=0; j<n; j++)  {
  for (k=0; k<n; k++) {
    r = B[k][j];
    for (i=0; i<n; i++)
      C[i][j] += A[i][k] * r;
  }
}
```

	A	B	C
I	?	?	?
J	?	?	?
K	?	?	?

Total Cache Misses (jki)

```
for (j=0; j<n; j++)  {
  for (k=0; k<n; k++) {
    r = B[k][j];
    for (i=0; i<n; i++)
      C[i][j] += A[i][k] * r;
  }
}
```

Matrices are very large compared to cache size

	A	**B**	**C**
I	n	1	n
J	n	n	n
K	n	n	n
	$\mathbf{n^3}$	$\mathbf{n^2}$	$\mathbf{n^3}$

Cache Miss Analysis for MVM

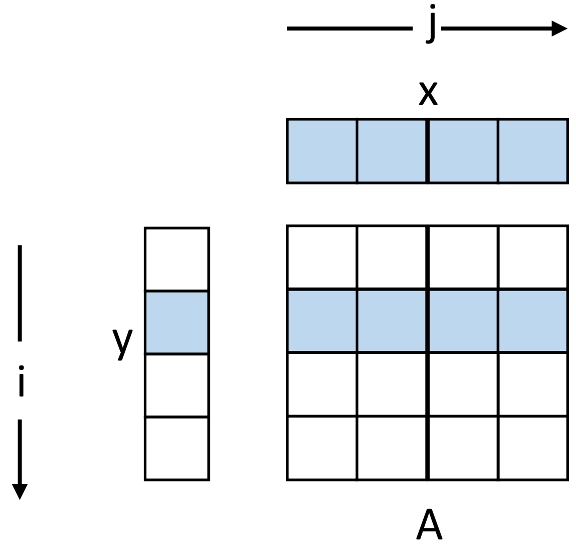
```
for (i = 0; i < M; i++) {
  for (j = 0; j < N; j++) {
    y[i] += A[i][j]*x[j];
  }
  return sum;
}
```

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 3 & 0 & 4 \\ 0 & 0 & 5 & 0 \\ 6 & 0 & 0 & 7 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 5 \\ 1 \\ 8 \end{bmatrix} = \begin{bmatrix} 4 \\ 47 \\ 5 \\ 68 \end{bmatrix}$$

- Number of memory locations: $N^2 + 2N$
- Number of operations: $O(N^2)$
- MVM is limited by memory bandwidth unlike matmul

MVM (ij)

```
for (i = 0; i < M; i++) {
  for (j = 0; j < N; j++)
    y[i] += A[i][j]*x[j];
  return sum;
}
```



Large Cache Model

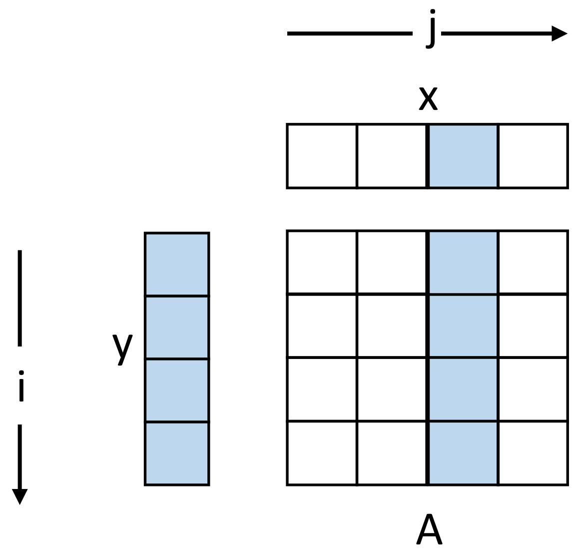
- Misses
  - A: N^2/B
  - X: N/B
  - Y: N/B
- Total: $N^2/B + 2N/B$

Small Cache Model

- Misses
  - A: N^2/B
  - X: N/B * N
  - Y: N/B
- Total: $2N^2/B + N/B$

MVM (ji)

```
for (j = 0; j < M; j++) {
  for (i = 0; i < N; i++)
    y[i] += A[i][j]*x[j];
  return sum;
}
```



Large Cache Model

- Misses
  - A: N^2/B
  - X: N/B
  - Y: N/B
- Total: $N^2/B + 2N/B$

Small Cache Model

- Misses
  - A: N^2
  - X: N/B
  - Y: N^2/B
- Total: $N^2 + N^2/B + N/B$

Cache Miss Estimation Example

- Consider a cache of size 64K words and block of size 8 words
- Perform cache miss analysis considering (i) direct-mapped and (ii) fully-associative caches

```
#define N 512
double A[N][N], B[N][N], C[N][N];

for (i = 0; i < N; i++)
  for (j = 0; j < N; j++)
    for (k = 0; k < N; k++)
      C[i][j] += A[i][k]*B[k][j];
```

Cache Miss Estimation Example

```
#define N 512
double A[N][N], B[N][N], C[N][N];

for (i = 0; i < N; i++)
  for (j = 0; j < N; j++)
    for (k = 0; k < N; k++)
      C[i][j] += A[i][k]*B[k][j];
```

Direct-mapped cache

	A	**B**	**C**
I	N	N	N
J	1	N	N/B
K	N/B	N	1
	N^2/B	**N^3**	**N^2/B**

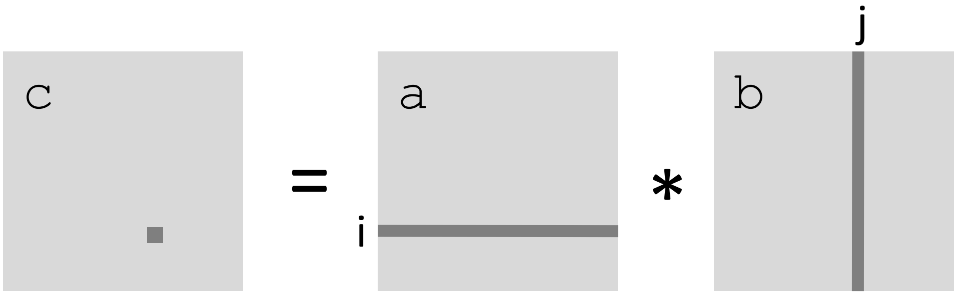
Fully-associative cache

	A	**B**	**C**
I	N	N	N
J	1	N/B	N/B
K	N/B	N	1
	N^2/B	**N^3/B**	**N^2/B**

Using Blocking to Improve Temporal Locality

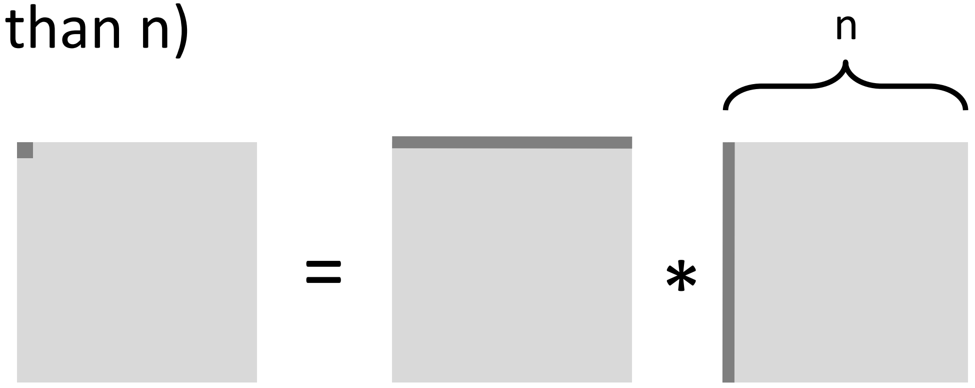
Revisit Matrix Multiplication

```
/* Multiply nxn matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
  for (int i = 0; i < n; i++)
      for (int j = 0; j < n; j++)
        for (int k = 0; k < n; k++)
          c[i*n+j] += a[i*n + k]*b[k*n + j];
}
```



Cache Miss Analysis

- Assume:
  - Matrix elements are doubles
  - Cache block = 8 doubles
  - Cache size << n (much smaller than n)



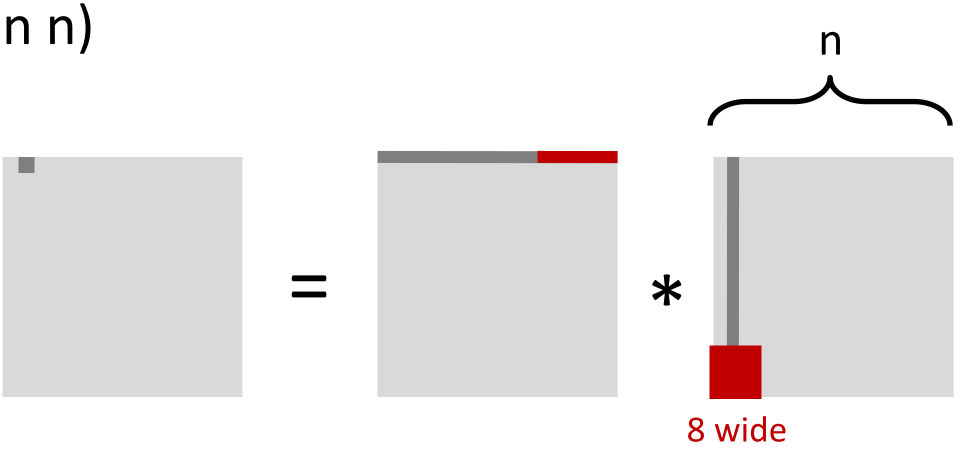
- First iteration:
  - $\frac{n}{8} + n = \frac{9n}{8}$ misses

  - Afterwards in cache: (schematic)

Cache Miss Analysis

- Assume:
  - Matrix elements are doubles
  - Cache block = 8 doubles
  - Cache size << n (much smaller than n)

- Second iteration:
  - $\frac{n}{8} + n = \frac{9n}{8}$ misses



- Total misses:
  - $\frac{9n}{8} * n^2 = \frac{9}{8} n^3$

Cache Blocking

- Improve data reuse by chunking the data in to smaller blocks
  - The block is supposed to fit in the cache

```
for (i = 0; i < N; i++) {
  …
}
```

```
for (j = 0; j < N; j +=BLK) {
  for (i = j; i < min(N, j+BLK); i++) {
    …
  }
}
```

```
for (body1=0; body1<NBODIES; body1++) {
  for (body2=0; body2<NBODIES; body2++) {
    OUT[body1] += compute(body1,body2);
  }
}
```

```
for (body2=0; body2<NBODIES; body2+=BLOCK) {
  for (body1=0; body1<NBODIES; body1++) {
    for (body22=0; body22<BLOCK; body22++)
      OUT[body1] += compute(body1,body2+body22);
  }
}
```

Cache Blocking Techniques

MVM with 2x2 Blocking

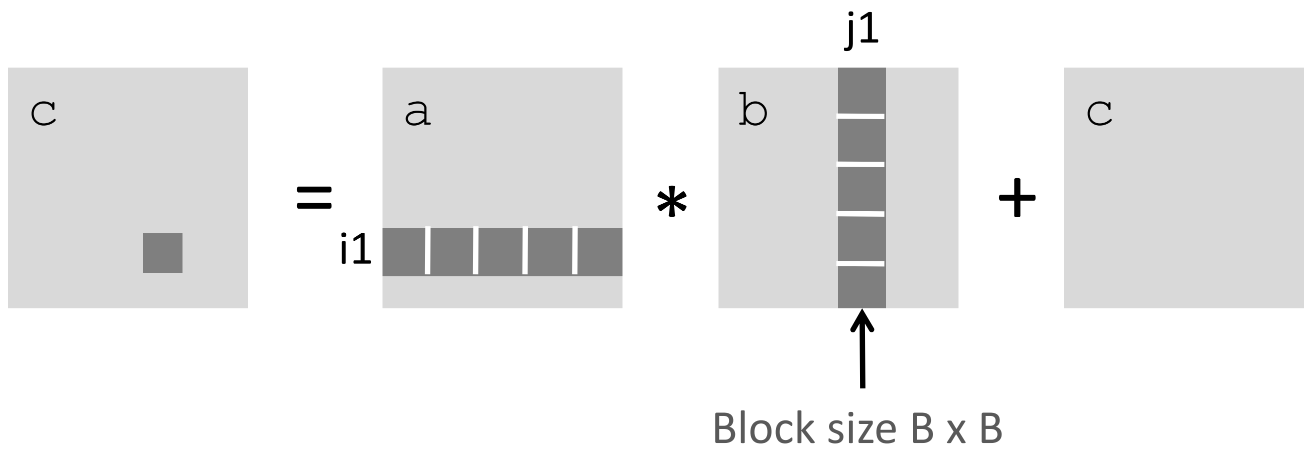
```
int i, j;
int a[100][100], b[100], c[100];
int n = 100;
for (i = 0; i < n; i++) {
  c[i] = 0;
  for (j = 0; j < n; j++) {
    c[i] = c[i] + a[i][j] * b[j];
  }
}
```

```
int i, j, x, y;
int a[100][100], b[100], c[100];
int n = 100;
for (i = 0; i < n; i += 2) {
  c[i] = 0; c[i + 1] = 0;
  for (j = 0; j < n; j += 2) {
    for (x = i; x < min(i + 2, n); x++) {
      for (y = j; y < min(j + 2, n); y++)
        c[x] = c[x] + a[x][y] * b[y];
    }
  }
}
```

Loop nest optimization

Blocked Matrix Multiplication

```
/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
  for (int i = 0; i < n; i+=BLK)
    for (int j = 0; j < n; j+=BLK)
      for (k = 0; k < n; k+=BLK)
        /* BxB mini-matrix (blocks) multiplications */
        for (int i1 = i; i1 < i+BLK; i++)
          for (int j1 = j; j1 < j+BLK; j++)
            for (int k1 = k; k1 < k+BLK; k++)
              c[i1*n+j1] += a[i1*n + k1]*b[k1*n + j1];
}
```



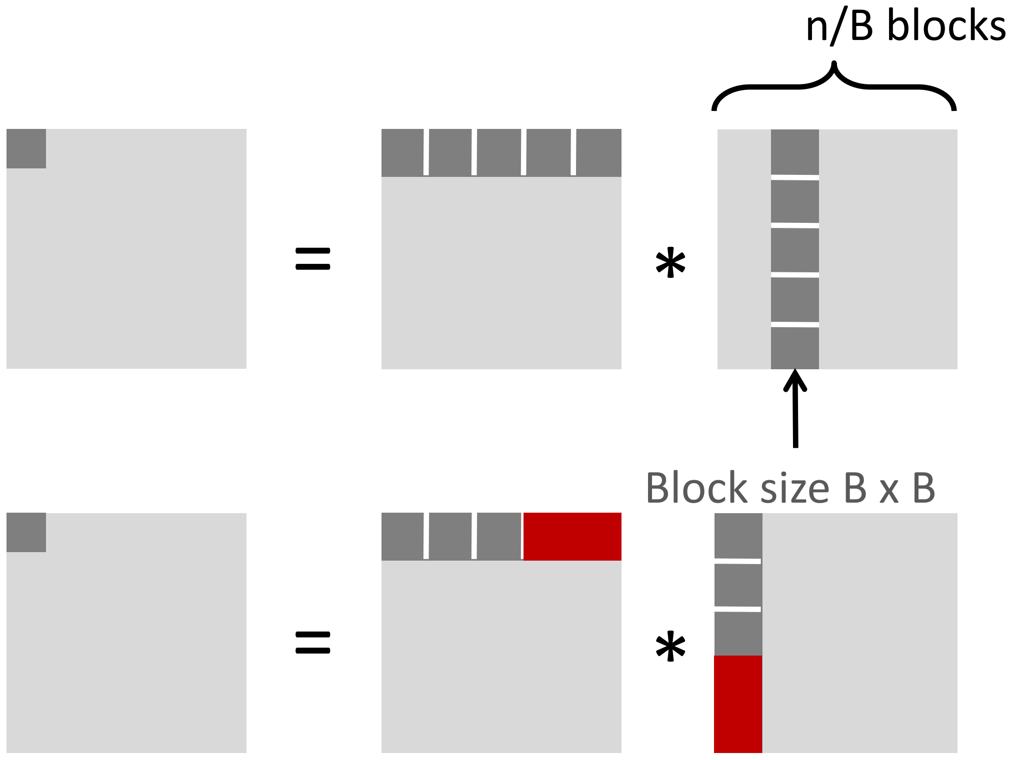
Cache Miss Analysis

- Assume:
  - Cache block = 8 doubles
  - Cache size << n (much smaller than n)
  - Three blocks ■ fit into cache, i.e., $3B^2 < C$

- First (block) iteration:
  - $\frac{B^2}{8}$ misses for each block
  - $2 * \frac{n}{B} * \frac{B^2}{8} = \frac{2*nB}{8}$

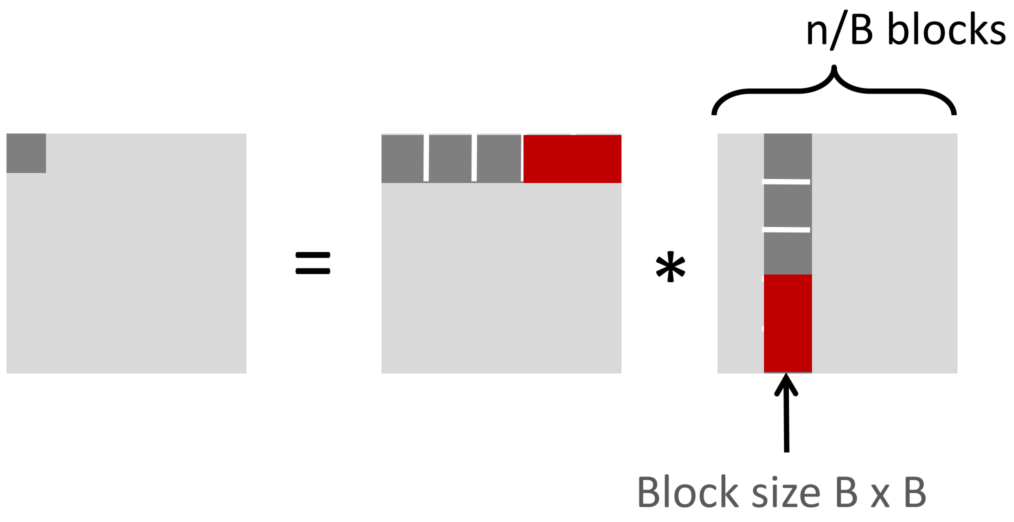
  (ignoring matrix C)

  - Afterwards in cache (schematic)



Cache Miss Analysis

- Assume:
  - Cache block = 8 doubles
  - Cache size << n (much smaller than n)
  - Three blocks ■ fit into cache, i.e., $3B^2 < C$
- Second (block) iteration:
  - Same as first iteration
  - $2 * \frac{n}{B} * \frac{B^2}{8} = \frac{2*nB}{8}$
- Total misses:
  - $\frac{2*nB}{8} * (\frac{n}{B})^2 = \frac{2*n^3}{8*B}$



Summary

Without blocking	With blocking
$\frac{9}{8} * n^3$	$\frac{2}{8B} * n^3$

- Find largest possible block size BLK, but limit $3BLK^2 < C$!
- Reason for dramatic difference is that matrix multiplication has inherent temporal locality
  - Input data is $3n^2$, computation is $2n^3$, and every array element is used $O(n)$ times!
  - But the program has to be written properly (choose good variant and BLK)

Pointers to Exploit Locality in your Code

Focus on the more frequently executed parts of the code (aka common case) (e.g., inner loops)

Maximize spatial locality with low strides (preferably 1)

Maximize temporal locality by reusing the data as much as possible

References

- Keshav Pingali – CS 377P: Programming for Performance, UT Austin.
- P. Sadayappan and A. Sukumaran Rajam – CS 5441: Parallel Computing, Ohio State University.
- R. Bryant and D. O’Hallaron – Cache Memories, CS 15-213, Introduction to Computer Systems., CMU.
- R. Bryant and D. O’Hallaron – Computer Systems: A Programmer’s Perspective.
- A. Aho et al. – Compilers: Principles, Techniques and Tools.