CS 610: Loop Transformations

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Content influenced by many excellent references, see References slide for acknowledgements.

Enhancing Program Performance

Fundamental issues

- Adequate fine-grained parallelism
  - Multiple pipelined functional units in each core
  - Exploit vector instruction sets (SSE, AVX, AVX-512)
- Adequate parallelism for SMP-type systems
  - Keep multiple asynchronous processors busy with work
- Minimize cost of memory accesses

Role of a Good Compiler

Try and extract performance automatically

Optimize memory access latency

- **Code restructuring optimizations**
- Prefetching optimizations
- Data layout optimizations
- Code layout optimizations

Loop Optimizations

- Loops are one of most commonly used constructs in HPC program
- Compiler performs many of loop optimization techniques automatically
  - In some cases source code modifications enhance optimizer's ability to transform code

Reordering Transformations

- A reordering transformation does not add or remove statements from a loop nest
    - Only reorders the execution of the statements that are already in the loop

Do not add or remove statements → Do not add or remove any new dependences

Reordering Transformations

- A reordering transformation does not add or remove statements from a loop nest
  - Only reorders the execution of the statements that are already in the loop

A reordering transformation is valid if it preserves all existing dependences in the loop

Iteration Reordering and Parallelization

- A transformation that reorders the iterations of a level-*k* loop, without making any other changes, is valid if the loop carries no dependence
- Each iteration of a loop may be executed in parallel if it carries no dependences

Data Dependence Graph and Parallelization

- If the Data Dependence Graph (DDG) is acyclic, then vectorization of the program is possible and is straightforward
- Otherwise, try to transform the DDG to an acyclic graph

Enhancing Fine-Grained Parallelism

Focus on Parallelization of Inner Loops

System Setup

- Setup: vector or superscalar architectures

- Focus is mostly on parallelizing the inner loops
- We will see optimizations for coarse-grained parallelism later

Loop Interchange (Loop Permutation)

- Switch the nesting order of loops in a **perfect** loop nest
- Can increase parallelism, can improve spatial locality

```
  DO I = 1, N
    DO J = 1, M
S     A(I,J+1) = A(I,J) + B
    ENDDO
  ENDDO
```

- Dependence is now carried by the outer loop
  - Inner-loop can be vectorized

```
  DO J = 1, M
    DO I = 1, N
S     A(I,J+1) = A(I,J) + B
    ENDDO
  ENDDO
```

Interchange of Non-rectangular Loops

```
for (i=0; i<n; i++)
  for (j=0; j<i; j++)
    y[i] = y[i] + A[i][j]*x[j];
```

???

Interchange of Non-rectangular Loops

```
for (i=0; i<n; i++)
  for (j=0; j<i; j++)
    y[i] = y[i] + A[i][j]*x[j];
```

```
for (j=0; j<n; j++)
  for (i=j+1; i<n; i++)
    y[i] = y[i] + A[i][j]*x[j];
```

Example of Loop Interchange

```
do i = 1, n
  do j = 1, n
    C(i, j) = C(i+1, j-1)
  enddo
enddo
```

```
do j = 1, n
  do i = 1, n
    C(i,j) = C(i+1,j-1)
  enddo
enddo
```

Valid?

Validity of Loop Interchange

1. Construct direction vectors for all possible dependences in the loop
   - Also called a direction matrix
   - Identical direction vectors are represented by a single row in the matrix
2. Compute direction vectors after permutation
3. Permutation of the loops in a perfect nest is legal iff there are no “-” direction as the leftmost non–“0” direction in any direction vector

Legality of Loop Interchange

(0, 0)

- Dependence is loop-independent

(0, +)

- Dependence is carried by the j^{th} loop, which remains the same after interchange

(+, 0)

- Dependence is carried by the i^{th} loop, relations do not change after interchange

(+, +)

- Dependence relations remain positive in both dimensions

(+, -)

- Dependence is carried by i^{th} loop, interchange results in an illegal direction vector

(0, +)

- Dependence is carried by the j^{th} loop, which remains the same after interchange

(0, -) (-, *)

- Such direction vectors are illegal, should not appear in the original loop

Validity of Loop Interchange

- Loop interchange is valid for a 2D loop nest if none of the dependence vectors has any negative components
- Interchange is legal: (1,1), (2,1), (0,1), (3,0)
- Interchange is not legal: (1,-1), (3,-2)

```
DO J = 1, M
  DO I = 1, N
    A(I,J+1) = A(I+1,J) + B
  ENDDO
ENDDO
```

Validity of Loop Permutation

- Generalization to higher-dimensional loops
- Permute all dependence vectors exactly the same way as the intended loop permutation
- If any permuted vector is lexicographically negative, permutation is illegal
- Example: d1 = (1,-1,1) and d2 = (0,2,-1)
  - ijk -> jik? (1,-1,1) -> (-1,1,1): illegal
  - ijk -> kij? (0,2,-1) -> (-1,0,2): illegal
  - ijk -> ikj? (0,2,-1) -> (0,-1,2): illegal
  - No valid permutation:
    - j cannot be outermost loop (-1 component in d1)
    - k cannot be outermost loop (-1 component in d2)

Valid or Invalid Loop Interchange?

```
DO I = 1, N
  DO J = 1, M
    DO K = 1, L
      A(I+1,J+1,K) = A(I,J,K) + A(I,J+1,K+1)
    ENDDO
  ENDDO
ENDDO
```

Benefits from Loop Permutation

```
for (i=0; i<n; i++)
  for (j=0; j<n; j++)
    for (k=0; k<n; k++)
      C[i][j] += A[i][k]*B[k][j];
```

Stride	ikj	kij	jik	ijk	jki	kji
C[i][j]	1	1	0	0	n	n
A[i][k]	0	0	1	1	n	n
B[k][j]	1	1	n	n	0	0

Understanding Loop Interchange

Pros	Cons
• Goal is to improve locality of reference or allow vectorization	• Need to careful about the iteration order, order of array accesses, and data involved

Does Loop Interchange/Permutation Always Help?

```
do i = 1, 10000
   do j = 1, 1000
     a[i] = a[i] + b[j,i] * c[i]
   end do
 end do
```

```
do I = 1, N
   do J = 1, M
     do K = 1, L
       A(I+1,J+1,K) = A(I,J,K) + B
     end do
   end do
end do
```

- Type and benefit from loop interchange depends on the target machine, the data structures accessed, memory layout and stride patterns
- Optimization choices for the snippet on the right
  - Vectorize J and K, Vectorize I assuming column-major layout, Parallelize K with threads

Loop Shifting

- In a perfect loop nest, if loops at level *i, i*+1,…, *i+n* carry no dependence—that is, all dependences are carried by loops at level less than *i* or greater than *i+n*—it is always legal to shift these loops inside of loop *i+n*+1.
- These loops will not carry any dependences in their new position.

Loops *i* to *i+n*

Dependence carried by outer loops

+	0	+	0	0	0
0	+	-	+	+	0
0	0	0	0	+	+
0	0	0	0	0	+

Dependence carried by inner loops

Loop Shift for Matrix Multiply

```
      DO I = 1, N
        DO J = 1, N
          DO K = 1, N
S           A(I,J) = A(I,J) + B(I,K)*C(K,J)
          ENDDO
        ENDDO
      ENDDO
```

Is the loop nest vectorizable as is?

We can move loops I and J inside

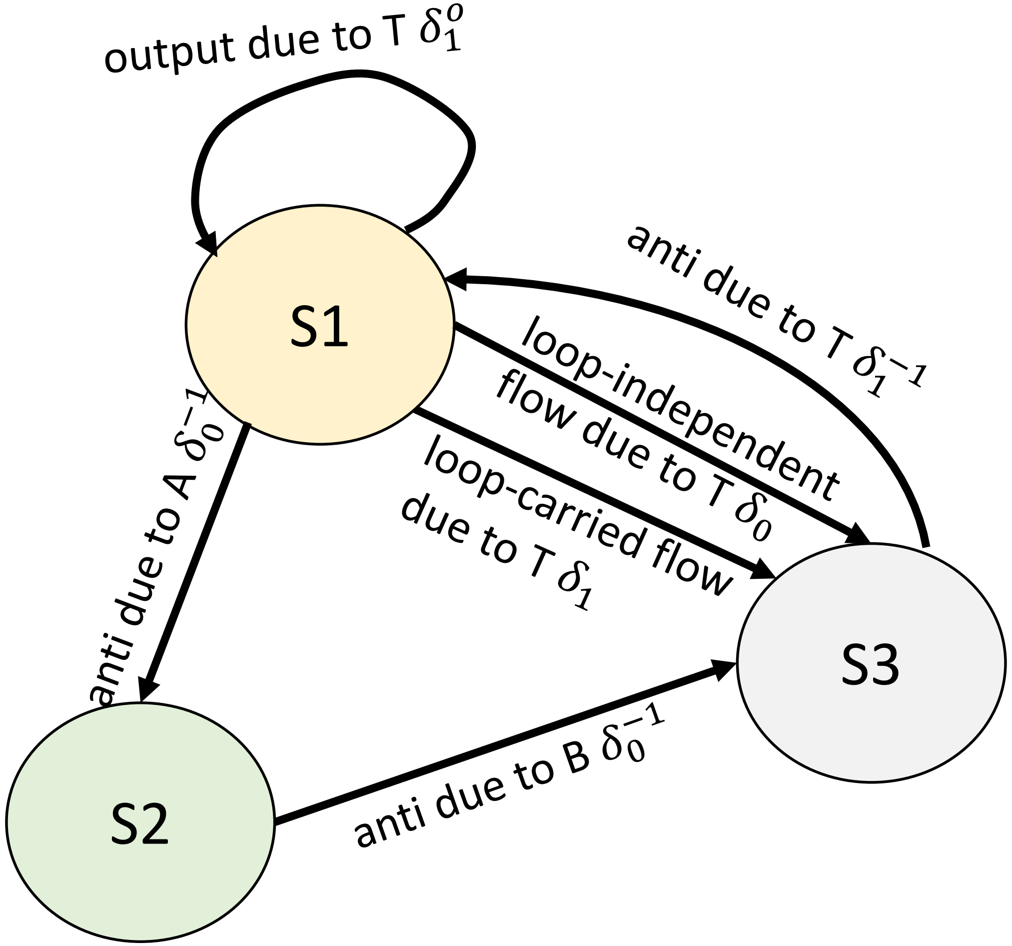
Loop Shift for Matrix Multiply

```
    DO I = 1, N
      DO J = 1, N
        DO K = 1, N
S         A(I,J) = A(I,J) + B(I,K)*C(K,J)
        ENDDO
      ENDDO
    ENDDO
```

```
    DO K = 1, N
      DO I = 1, N
        DO J = 1, N
S         A(I,J) = A(I,J) + B(I,K)*C(K,J)
        ENDDO
      ENDDO
    ENDDO
```

Scalar Expansion

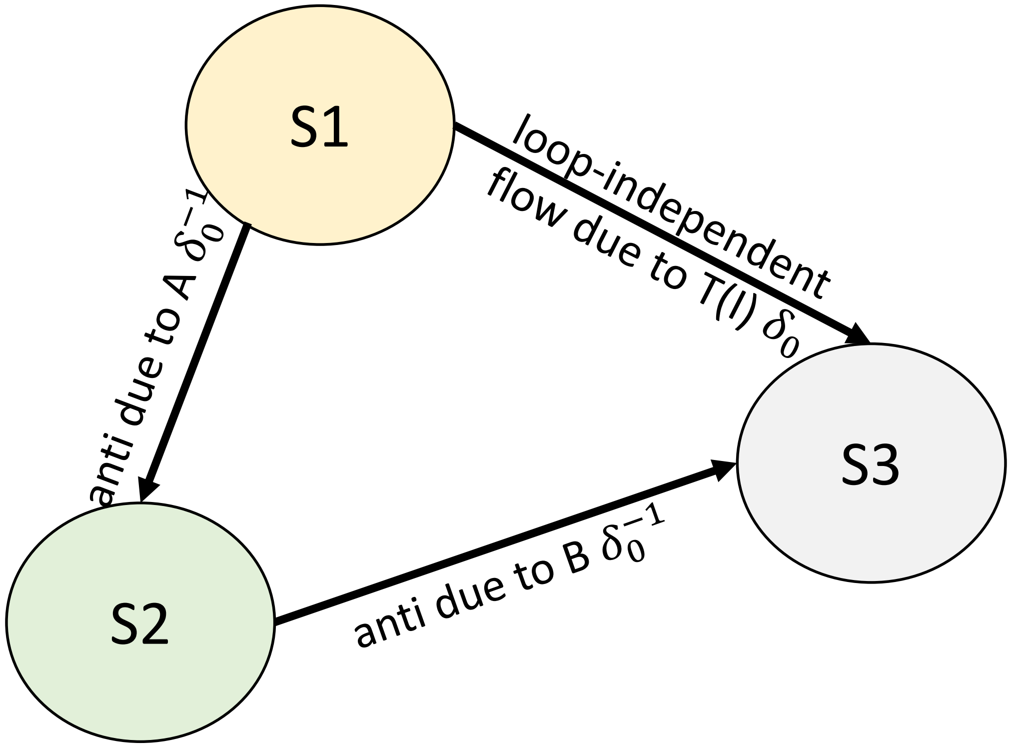
```
    DO I = 1, N
S1    T = A(I)
S2    A(I) = B(I)
S3    B(I) = T
    ENDDO
```



Scalar Expansion

```
    DO I = 1, N
S1    $T(I) = A(I)
S2    A(I) = B(I)
S3    B(I) = $T(I)
    ENDDO
    T = $T(N)
```

Eliminate dependences that arise from reuse of memory locations at the cost of extra memory



Scalar Expansion

```
DO I = 1, N
  T = T + A(I) + A(I-1)
  A(I) = T
ENDDO
```

Can we parallelize the I loop?
Check the dependence graph.

```
$T(0) = T
DO I = 1, N
  $T(I) = $T(I-1) + A(I) + A(I-1)
  A(I) = $T(I)
ENDDO
T = $T(N)
```

Understanding Scalar Expansion

Pros	Cons
• Eliminates dependences due to reuse of memory locations • Helps with uncovering parallelism	• Increases memory overhead • Complicates addressing

```
DO I = 1, N
  T = A(I) + A(I+1)
  A(I) = T + B(I)
ENDDO
```

Can also try forward substitution

➡

Strip-mining

```
DO I = 1, N, 64
  DO i = 0, 63
    T = A(I+i) + A(I+1+i)
  A(I+i) = T + B(I+i)
ENDDO
```

➡

Strip loop

```
DO I = 1, N, 64
  DO i = 0, 63
    $T(i) = A(I+i) + A(I+1+i)
  A(I+i) = $T(i) + B(I+i)
ENDDO
```

Limits of Scalar Expansion

```
    DO I = 1, 100
S1    T = A(I) + B(I)
S2    C(I) = T + T
S3    T = D(I) - B(I)
S4    A(I+1) = T * T
    ENDDO
```

```
    DO I = 1, 100
S1    $T(I) = A(I) + B(I)
S2    C(I) = $T(I) + $T(I)
S3    $T(I) = D(I) - B(I)
S4    A(I+1) = $T(I) * $T(I)
    ENDDO
```

Can we vectorize this loop nest? Draw the dependence graphs to check.

Scalar Renaming

```
     DO I = 1, 100
S1     T = A(I) + B(I)
S2     C(I) = T + T
S3     T = D(I) - B(I)
S4     A(I+1) = T * T
     ENDDO
```

```
     DO I = 1, 100
S1     T1 = A(I) + B(I)
S2     C(I) = T1 + T1
S3     T2 = D(I) - B(I)
S4     A(I+1) = T2 * T2
     ENDDO
     T = T2
```

Can we vectorize this loop nest given as is?

Allow Vectorization with Statement Interchange

```
    DO I = 1, 100
S1    T1 = A(I) + B(I)
S2    C(I) = T1 + T1
S3    T2 = D(I) - B(I)
S4    A(I+1) = T2 * T2
    ENDDO
    T = T2
```

➡

```
    DO I = 1, 100
S3    T2 = D(I) - B(I)
S4    A(I+1) = T2 * T2
S1    T1 = A(I) + B(I)
S2    C(I) = T1 + T1
    ENDDO
    T = T2
```

➡

```
S3    T2[1:100] = D(1:100) - B(1:100)
S4    A[2:101] = T2[1:100] * T2[1:100]
S1    T1[1:100] = A[1:100] + B[1:100]
S2    C[1:100] = T1[1:100] + T1[1:100]

    T = T2[100]
```

Array Renaming

```
    DO I = 1, 100
S1    A(I) = A(I-1) + X
S2    Y(I) = A(I) + Z
S3    A(I) = B(I) + C
    ENDDO
```

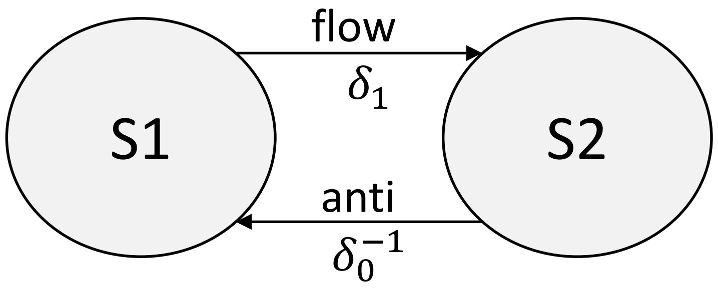
```
    DO I = 1, 100
S1    $A(I) = A(I-1) + X
S2    Y(I) = $A(I) + Z
S3    A(I) = B(I) + C
    ENDDO
```

Supporting array renaming requires sophisticated analysis

Node Splitting

```
    DO I = 1, 100
S1    A(I) = X(I+1) + X(I)
S2    X(I+1) = B(I) + 10
    ENDDO
```

```
    DO I = 1, 100
S0    $X(I) = X(I+1)
S1    A(I) = $X(I) + X(I)
S2    X(I+1) = B(I) + 10
    ENDD0
```



Index-Set Splitting

```
DO I = 1, 100
  A(I+20) = A(I) + B
ENDDO
```

```
DO I = 1, 100, 20
  DO i = I, I+19
    A(i+20) = A(i) + B
  ENDDO
ENDDO
```

strip-mining

An index-set splitting transformation subdivides the loop into different iteration ranges

Loop Peeling

- Splits any problematic iterations from the loop body
  - Could be first, middle, or last few iterations
- Change from a loop-carried dependence to loop-independent dependence
- Transformed loop carries no dependence, can be parallelized

```
DO I = 1, N
  A(I) = A(I) + A(1)
ENDDO
```

```
A(1) = A(1) + A(1)
DO I = 2, N
  A(I) = A(I) + A(1)
ENDDO
```

Loop Peeling

- Splits any problematic iterations from the loop body
  - Could be first, middle, or last few iterations
- Change from a loop-carried dependence to loop-independent dependence

```
int p = 10;
for (int i = 0; i < 10; ++i) {
  y[i] = x[i] + x[p];
  p = i;
}
```

```
y[0] = x[0] + x[10];
for (int i = 1; i < 10; ++i) {
  y[i] = x[i] + x[i-1];
}
```

https://en.wikipedia.org/wiki/Loop_splitting

Loop Splitting

assume N is divisible by 2

```
DO I = 1, N
  A(I) = A(N/2) + B(I)
ENDDO
```

```
M = N/2
DO I = 1, M-1
  A(I) = A(N/2) + B(I)
ENDDO

A(M) = A(N/2) + B(I)

DO I = M+1, N
  A(I) = A(N/2) + B(I)
ENDDO
```

Section-Based Splitting

```
    DO I = 1,N
      DO J = 1, N/2
S1      B(J,I) = A(J,I) + C
      ENDDO
      DO J = 1,N
S2      A(J,I+1) = B(J,I) + D
      ENDDO
    ENDDO
```

δ_1^{-1} δ_0

```
    DO I = 1,N
      DO J = 1, N/2
S1      B(J,I) = A(J,I) + C
      ENDDO
      DO J = 1,N/2
S2      A(J,I+1) = B(J,I) + D
      ENDDO
      DO J = N/2+1, N
S3      A(J,I+1) = B(J,I) + D
      ENDDO
    ENDDO
```

S3 is independent

Enabling Vectorization with Section-Based Splitting

```
    DO I = 1,N
      DO J = 1, N/2
S1      B(J,I) = A(J,I) + C
      ENDDO
      DO J = 1,N/2
S2      A(J,I+1) = B(J,I) + D
      ENDDO
      DO J = N/2+1, N
S3      A(J,I+1) = B(J,I) + D
      ENDDO
    ENDDO
```

```
    DO I = 1,N
      DO J = N/2+1, N
S3      A(J,I+1) = B(J,I) + D
      ENDDO
    DO I = 1,N
      DO J = 1,N/2
S1      B(J,I) = A(J,I) + C
      ENDDO
      DO J = 1, N/2
S2      A(J,I+1) = B(J,I) + D
      ENDDO
    ENDDO
```

Enabling Vectorization with Section-Based Splitting

```
    DO I = 1,N
      DO J = N/2+1, N
S3      A(J,I+1) = B(J,I) + D
      ENDDO
    DO I = 1,N
      DO J = 1,N/2
S1      B(J,I) = A(J,I) + C
      ENDDO
      DO J = 1, N/2
S2      A(J,I+1) = B(J,I) + D
      ENDDO
    ENDDO
```

cannot vectorize I

```
    M = N/2
S3  A(M+1:N,2:N+1) = B(M+1:N,1:N) + D
    DO I = 1, N
S1    B(1:M,I) = A(1:M,I) + C
S2    A(1:M,I+1) = B(1:M,I) + D
    ENDDO
```

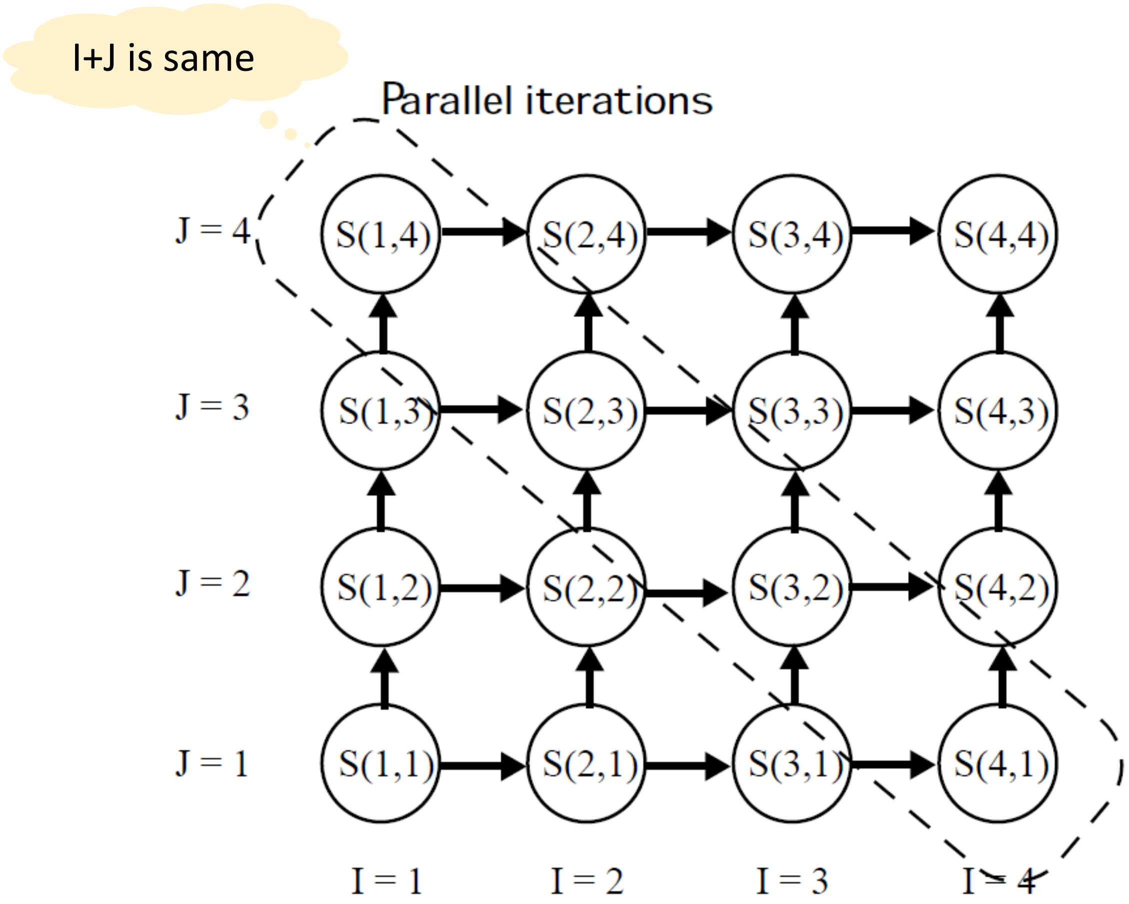
Draw the Dependence Graph

```
    DO I = 1, N
      DO J = 1, N
S       A(I,J) = A(I-1,J) + A(I,J-1)
      ENDDO
    ENDDO
```

Which loops carry dependences?

Loop Skewing

```
    DO I = 1, N
      DO J = 1, N
S       A(I,J) = A(I-1,J) + A(I,J-1)
      ENDDO
    ENDDO
```

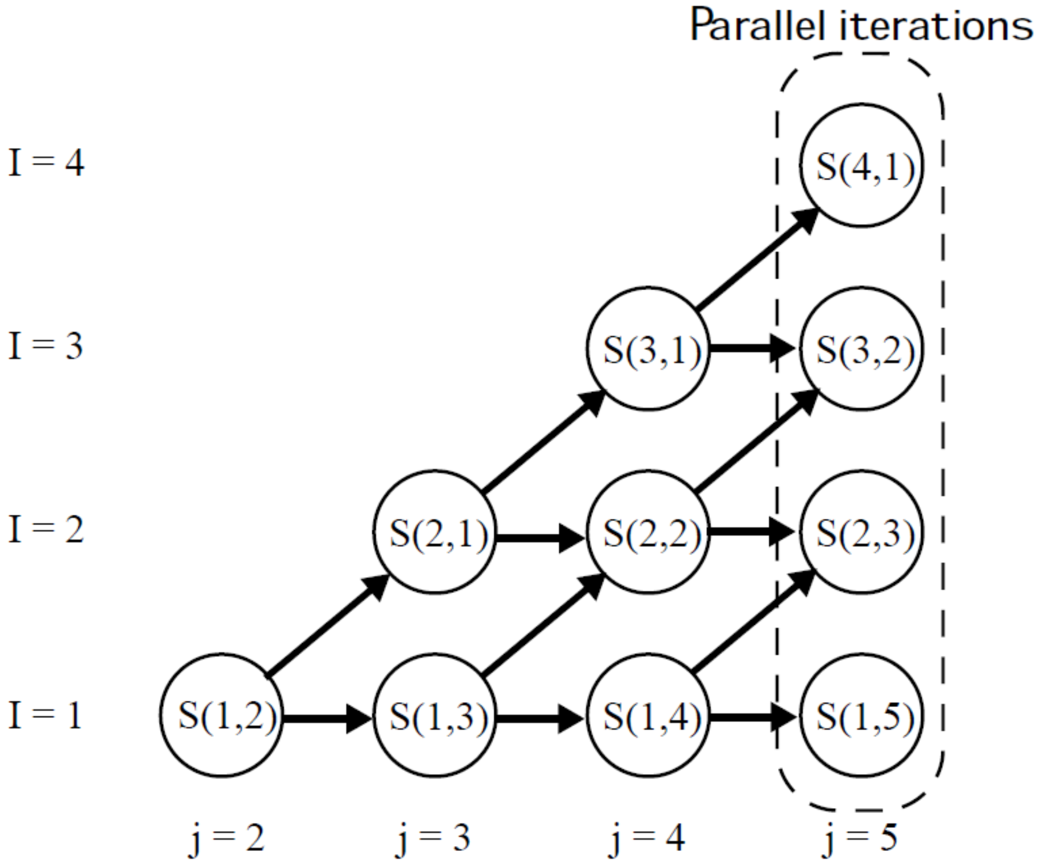


Loop Skewing

j = I+J

```
    DO I = 1, N
      DO j = I+1, I+N
S       A(I,j-I) = A(I-1,j-I) + A(I,j-I-1)
      ENDDO
    ENDDO
```

- What are the dependences now?
- Which loop carries the dependence?



Perform Loop Interchange

```
   DO I = 1, N
     DO j = I+1, I+N
S      A(I,j-I) = A(I-1,j-I) + A(I,j-I-1)
     ENDDO
   ENDDO
```

Are the loops vectorizable?

⬇

```
   DO j = 2, N+N
     DO I = max(1,j-N), min(N,j-1)
S      A(I,j-I) = A(I-1,j-I) + A(I,j-I-1)
     ENDDO
   ENDDO
```

Use Fourier-Motzkin elimination

Understanding Loop Skewing

Pros	Cons
• Reshapes the iteration space to find possible parallelism • Allows for loop interchange in future	• Resulting iteration space can be trapezoidal • Irregular loops are not very amenable for vectorization • Need to be careful about load imbalance

Loop Unrolling (Loop Unwinding)

```
for (i = 0; i < n; i++) {
  a[i] = a[i-1] + a[i] + a[i+1];
}
```

```
for (i = 0; i < n; i += 4) {
  a[i] = a[i-1] + a[i] + a[i+1];
  a[i+1] = a[i] + a[i+1] + a[i+2];
  a[i+2] = a[i+1] + a[i+2] + a[i+3];
  a[i+3] = a[i+2] + a[i+3] + a[i+4];
}
int f = n % 4;
for (i = n - f; i < n; i++) {
  a[i] = a[i-1] + a[i] + a[i+1];
}
```

Loop Unrolling (Loop Unwinding)

- Reduce number of iterations of loops
  - Add statement(s) to do work of missing iterations
- JIT compilers try to perform unrolling at run-time

```
for (i = 0; i < n; i++) {
  for (j = 0; j < 2*m; j++) {
    loop-body(i, j);
  }
}
```

```
for (i = 0; i < n; i++) {
  for (j = 0; j < 2*m; j+=2) {
    loop-body(i, j);
    loop-body(i, j+1);
  }
}
```

2-way unrolled

Inner Loop Unrolling

```
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
    y[i] = y[i] + a[i][j]*x[j];
  }
}
```

```
for (i=0; i<n; i++) {
  for (j=0; j<n; j+=4) {
    y[i] = y[i] + a[i][j]*x[j];
    y[i] = y[i] + a[i][j+1]*x[j+1];
    y[i] = y[i] + a[i][j+2]*x[j+2];
    y[i] = y[i] + a[i][j+3]*x[j+3];
  }
}
```

Inner Loop Unrolling

```
for (i=0; i<n; i++) {
  for (j=0; j<n; j+=4) {
    y[i] = y[i] + a[i][j]*x[j];
    y[i] = y[i] + a[i][j+1]*x[j+1];
    y[i] = y[i] + a[i][j+2]*x[j+2];
    y[i] = y[i] + a[i][j+3]*x[j+3];
  }
}
```

```
for (i=0; i<n; i++) {
  for (j=0; j<n; j+=4) {
    y[i] = y[i] + a[i][j]*x[j]
                + a[i][j+1]*x[j+1]
                + a[i][j+2]*x[j+2]
                + a[i][j+3]*x[j+3];
  }
}
```

Outer Loop Unrolling + Inner Loop Jamming

```
for (i=0; i<2*n; i++) {
  for(j=0; j<m; j++) {
    loop-body(i,j);
  }
}
```

```
for (i=0; i<2*n; i+=2) {
  for(j=0; j<m; j++) {
    loop-body(i,j)
  }
  for(j=0; j<m; j++) {
    loop-body(i+1,j)
  }
}
```

```
for (i=0; i<2*n; i+=2) {
  for(j=0; j<m; j++) {
    loop-body(i,j)
    loop-body(i+1,j)
  }
}
```

2-way outer unroll does not increase operation-level parallelism

Is Unroll and Jam Legal?

```
DO I = 1, N
  DO J = 1, M
    A(I,J) = A(I-1,J+1)+C
  ENDDO
ENDDO
```

```
DO I = 1, N, 2
  DO J = 1, M
    A(I,J) = A(I-1,J+1)+C
    A(I+1,J) = A(I,J+1)+C
  ENDDO
ENDDO
```

Validity Condition for Loop Unroll/Jam

- Sufficient condition can be obtained by observing that complete unroll/jam of a loop is equivalent to a loop permutation that moves that loop innermost, without changing order of other loops
- If such a loop permutation is valid, unroll/jam of the loop is valid
- Example: 4D loop `ijkl`; d1 = (1,-1,0,2), d2 = (1,1,-2,-1)
  - `i`: d1-> (-1,0,2,**1**) => invalid to unroll/jam
  - `j`: d1-> (1,0,2,**-1**); d2 -> (1,-2,-1,**1**) => valid to unroll/jam
  - `k`: d1 -> (1,-1,2,**0**); d2 -> (1,1,-1,**-2**) => valid to unroll/jam
  - `l`: d1 and d2 are unchanged; innermost loop always unrollable

Understanding Loop Unrolling

Pros	Cons
• Small loop bodies are problematic, reduces control overhead of loops • Increases operation-level parallelism in loop body • Allows other optimizations like reuse of temporaries across iterations	• Increases the executable size • Increases register usage • May prevent function inlining

Loop Tiling

- Improve data reuse by chunking the data in to smaller blocks (tiles)
  - The block is supposed to fit in the cache
- Tries to exploit spatial and temporal locality of data

```
for (i = 0; i < N; i++) {
  …
}
```

```
for (j = 0; j < N; j +=B) {
  for (i = j; i < min(N, j+B); j++) {
    …
  }
}
```

MVM with 2x2 Blocking

```
int i, j, a[100][100], b[100], c[100];
int n = 100;
for (i = 0; i < n; i++) {
  c[i] = 0;
  for (j = 0; j < n; j++) {
    c[i] = c[i] + a[i][j] * b[j];
  }
}
```

```
int i, j, x, y, a[100][100], b[100], c[100];
int n = 100;
for (i = 0; i < n; i += 2) {
  c[i] = 0;
  c[i + 1] = 0;
  for (j = 0; j < n; j += 2) {
    for (x = i; x < min(i + 2, n); x++) {
      for (y = j; y < min(j + 2, n); y++) {
        c[x] = c[x] + a[x][y] * b[y];
      }
    }
  }
}
```

https://en.wikipedia.org/wiki/Loop_nest_optimization

Loop Tiling

- Determining the tile size
  - Requires accurate estimate of array accesses and the cache size of the target machine
  - Loop nest order also influences performance
  - Difficult theoretical problem, usually heuristics are applied
  - Cache-oblivious algorithms make efficient use of cache without explicit blocking

Validity Condition for Loop Tiling

- A contiguous band of loops can be tiled if they are fully permutable
- A band of loops is fully permutable if all permutations of the loops in that band are legal
- Example: d = (1,2,-3)
  - Tiling all three loops `ijk` is not valid, since the permutation `kij` is invalid
  - 2D tiling of band `ij` is valid
  - 2D tiling of band `jk` is valid

```
for (i = 0; i < n; i++)
  for (j = 0; j < n; j++)
    for (k = 0; k < n; k++)
      loop_body(i,j,k)
```

⬇

```
for (it = 0; it < n; it+=T)
  for (jt = 0; tj < n; j+=T)
    for (i = it; i < it+T; i++)
      for (j = jt; j < jt+T; j++)
        for (k = 0; k < n; k++)
          loop_body(i,j,k)
```

Creating Coarse-Grained Parallelism

Find Work For Threads

- Setup
  - Symmetric multiprocessors with shared-memory
  - Threads are running on each core and are coordinating execution with occasional synchronization

  - A basic synchronization element is a barrier
  - A barrier in a program forces all processes to reach a certain point before execution continues

- Challenge: Balance the granularity of parallelism with communication overheads

Challenges in Coarse-Grained Parallelism

Minimize communication and synchronization overhead while evenly load balancing across the processors

- Running everything on one processor achieves minimal communication and synchronization overhead
- Very fine-grained parallelism achieves good load balance, but benefits may be outweighed by frequent communication and synchronization

Challenges in Coarse-Grained Parallelism

Minimize communication an[...] while evenly
load balancin[...]

One expectation from an optimizing compiler is to find the sweet spot

- Running[...] achieves
  achieves[...]nefits may
  synchroni[...]equent
  [...]nication and synchronization

Few Ideas to Try

- Single loop
  - Carries a dependence ➔ Try transformations to eliminate the loop-carried dependence
    - For example, loop distribution and scalar expansion
  - Decide on the granularity of the new parallel loop
- Perfect loop nests
  - Try loop interchange to see if the dependence level can be changed

Privatization

- Privatization is similar in flavor to scalar expansion
- Temporaries can be made local to each iteration

```
    DO I = 1,N
S1     T = A(I)
S2     A(I) = B(I)
S3     B(I) = T
    ENDDO
```

```
    PARALLEL DO I = 1,N
      PRIVATE t
S1    t = A(I)
S2    A(I) = B(I)
S3    B(I) = t
    ENDDO
```

Privatization

- A scalar variable x in a loop L is **privatizable** if every path from the entry of L to a use of x in the loop passes through a definition of x
  - No use of the variable is upward exposed, i.e., the use never reads a value that was assigned outside the loop
  - No use of the variable is from an assignment in an earlier iteration
- Computing upward-exposed variables from a block x

$$up(x) = use(x) \cup (\neg\, def(x) \ \cap \bigcup_{y \in succ(x)} up(y))$$

- Computing privatizable variables for a loop body B where b_0 is the entry block

$$private(B) = \neg up(b_0) \cap (\bigcup_{y \in B} def(y))$$

Privatization

- If all dependences carried by a loop involve a privatizable variable, then loop can be parallelized by making the variables private
- Preferred compared to scalar expansion
  - Less memory requirement
  - Scalar expansion may suffer from false sharing
- However, there can be situations where scalar expansion works but privatization does not

Comparing Privatization and Scalar Expansion

```
DO I = 1, N
  T = A(I) + B(I)
  A(I-1) = T
ENDDO
```

```
PARALLEL DO I = 1, N
  T$(I) = A(I) + B(I)
  A(I-1) = T$(I)
ENDDO
```

```
DO I = 1, N
  PRIVATE T
  T = A(I) + B(I)
  A(I-1) = T
ENDDO
```

```
PARALLEL DO I = 1, N
  T$(I) = A(I) + B(I)
ENDDO

PARALLEL DO I = 1, N
  A(I-1) = T$(I)
ENDDO
```

Loop Distribution (Loop Fission)

```
    DO I = 1, 100
      DO J = 1, 100
S1      A(I,J) = B(I,J) + C(I,J)
S2      D(I,J) = A(I,J-1) * 2.0
      ENDDO
    ENDDO
```

```
    DO I = 1, 100
      DO J = 1, 100
S1      A(I,J) = B(I,J) + C(I,J)
      ENDDO

      DO J = 1, 100
S2      D(I,J) = A(I,J-1) * 2.0
      ENDDO
    ENDDO
```

Eliminates loop-carried dependences

Validity Condition for Loop Distribution

- Sufficient (but not necessary) condition: A loop with two statements can be distributed if there are no dependences from any instance of the **later** statement to any instance of the **earlier** one
  - Generalizes to more statements

Validity Condition for Loop Distribution

- Example: Loop distribution is not valid (executing all S1 first and then all S2)

```
    For I = 1, N
S1    A(I) = B(I) + C(I)
S2    E(I) = A(I+1) * D(I)
    EndFor
```

- Example: Loop distribution is valid

```
    For I = 1, N
S1    A(I) = B(I) + C(I)
S2    E(I) = A(I-1) * D(I)
    EndFor
```

Understanding Loop Distribution

Pros	Cons
• Execute source of a dependence **before** the sink • Reduces the memory footprint of the original loop • For both data and code	• Can increase the synchronization required between dependence points

Loop Alignment

- Unlike loop distribution, realign the loop to compute and use the values in the same iteration

Cannot be parallelized

```
    DO I = 2, N
S1    A(I) = B(I) + C(I)
S2    D(I) = A(I-1) * 2.0
    ENDDO
```

```
    DO i = 1, N+1
      if i > 1 && i < N+1
S1      A(i) = B(i) + C(i)
      if i < N
S2      D(i+1) = A(i) * 2.0
    ENDDO
```

carried dependence becomes a loop independent

More on Loop Alignment

```
    DO I = 1, N
S1    A(I) = B(I) + C
S2    B(I+1) = A(I) + D
    ENDDO
```

A is aligned, B is misaligned

```
    DO i = 1, N+1
      if i > 1
S1      B(i) = A(i-1) + D
      if i < N+1
S2      A(i) = B(i) + C
    ENDDO
```

```
    DO I = 1, N
S1    A(I+1) = B(I) + C
S2    X(I) = A(I+1) + A(I)
    ENDDO
```

```
    DO i = 0, N
      if i > 0
S1      A(i+1) = B(i) + C
      if i < N
S2      X(i+1) = A(i+2) + A(i+1)
    ENDDO
```

Loop Fusion (Loop Jamming)

```
    DO I = 1, N
S1    A(I) = B(I) + 1
S2    C(I) = A(I) + C(I-1)
S3    D(I) = A(I) + X
    ENDDO
```

loop-carried dependence

```
L1  DO I = 1, N
      A(I) = B(I) + 1
    ENDDO
L2  DO I = 1, N
      C(I) = A(I) + C(I-1)
    ENDDO
L3  DO I = 1, N
      D(I) = A(I) + X
    ENDDO
```

Loop Fusion (Loop Jamming)

```
L1  DO I = 1, N
      A(I) = B(I) + 1
    ENDDO
L2  DO I = 1, N
      C(I) = A(I) + C(I-1)
    ENDDO
L3  DO I = 1, N
      D(I) = A(I) + X
    ENDDO
```

```
L13 PARALLEL DO I = 1, N
      A(I) = B(I) + 1
      D(I) = A(I) + X
    ENDDO
L2  DO I = 1, N
      C(I) = A(I) + C(I-1)
    ENDDO
```

Validity Condition for Loop Fusion

- Consider a loop-independent dependence between statements in two different loops (i.e., from S1 to S2)
- A dependence is fusion-preventing if fusing the two loops causes the dependence to be carried by the combined loop in the reverse direction (from S2 to S1)

```
    DO I = 1, N
S1    A(I) = B(I) + C
    ENDDO
    DO I = 1, N
S2    D(I) = A(I+1) + E
    ENDDO
```

Loop independent flow dependence

➡

```
    DO I = 1, N
S1    A(I) = B(I) + C
S2    D(I) = A(I+1) + E
    ENDDO
```

Backward loop-carried anti dependence

No

Understanding Loop Fusion

Pros	Cons
• Reduce overhead of loops • May improve temporal locality	• May decrease data locality in the fused loop

```
    DO I = 1, N
S1    A(I) = B(I) + C
    ENDDO
    DO I = 1, N
S2    D(I) = A(I-1) + E
    ENDDO
```

➡

```
    DO I = 1, N
S1    A(I) = B(I) + C
S2    D(I) = A(I-1) + E
    ENDDO
```

Yes

Loop Interchange

```
DO I = 1, N
  DO J = 1, M
    A(I+1,J) = A(I,J) + B(I,J)
  ENDDO
ENDDO
```

Loop I carries a dependence

Parallelizing J is good for vectorization, but not from coarse-grained parallelism

Loop Interchange

```
DO I = 1, N
  DO J = 1, M
    A(I+1,J) = A(I,J) + B(I,J)
  ENDDO
ENDDO
```

➡

```
DO J = 1, M
  DO I = 1, N
    A(I+1,J) = A(I,J) + B(I,J)
  ENDDO
ENDDO
```

⬇

```
PARALLEL DO J = 1, M
  DO I = 1, N
    A(I+1,J) = A(I,J) + B(I,J)
  ENDDO
END PARALLEL DO
```

Dependence-free loops should move to the outermost level

Loop Interchange

Vectorization	Coarse-grained Parallelism
• Move dependence-free loops to innermost level	• Move dependence-free loops to outermost level

Condition for Loop Interchange

- In a perfect loop nest, a loop can be parallelized at the outermost level if and only if the column of the direction matrix for that nest contains only “0” entries

```
DO I = 1, N
  DO J = 1, M
    A(I+1,J+1) = A(I,J) + B(I,J)
  ENDDO
ENDDO
```

Code Generation Strategy

1. Continue till there are no more columns to move
   - Choose a loop from the direction matrix that has all “0” entries in the column
   - Move it to the outermost position
   - Eliminate the column from the direction matrix
2. Pick loop with **most “+” entries**, move to the next outermost position
   - Generate a sequential loop
   - Eliminate the column
   - Eliminate any rows that represent dependences carried by this loop
3. Repeat from Step 1

Code Generation Example

```
DO I = 1, N
  DO J = 1, M
    DO K = 1, L
      A(I+1,J,K) = A(I,J,K) + X1
      B(I,J,K+1) = B(I,J,K) + X2
      C(I+1,J+1,K+1) = C(I,J,K) + X3
    ENDDO
  ENDDO
ENDDO
```

- What is the direction matrix?
- Can we permute the loops?

Code Generation Example

```
DO I = 1, N
  DO J = 1, M
    DO K = 1, L
      A(I+1,J,K) = A(I,J,K) + X1
      B(I,J,K+1) = B(I,J,K) + X2
      C(I+1,J+1,K+1) = C(I,J,K) + X3
    ENDDO
  ENDDO
ENDDO
```

+	0	0
0	0	+
+	+	+

Since there are no columns with all "0" entries, none of the loops can be parallelized at the outermost level

Generated Code

```
DO I = 1, N
  PARALLEL DO J = 1, M
    DO K = 1, L
      A(I+1,J,K) = A(I,J,K) + X1
      B(I,J,K+1) = B(I,J,K) + X2
      C(I+1,J+1,K+1) = C(I,J,K) + X3
    ENDDO
  END PARALLEL DO
ENDDO
```

How did we pick loop J?

+	0	0
0	**0**	**+**
+	+	+

How can we parallelize this loop?

```
DO I = 2, N+1
  DO J = 2, M+1
    DO K = 1, L
      A(I,J,K) = A(I,J-1,K+1) + A(I-1,J,K+1)
    ENDDO
  ENDDO
ENDDO
```

0	**+**	**-**
+	**0**	**-**

No single loop carries all the dependences, so we can only parallelize loop K

Loop Reversal

```
DO I = 2, N+1
  DO J = 2, M+1
    DO K = 1, L
      A(I,J,K) = A(I,J-1,K+1) + A(I-1,J,K+1)
    ENDDO
  ENDDO
ENDDO
```

0	+	-
+	0	-

```
DO I = 2, N+1
  DO J = 2, M+1
    DO K = L, 1, -1
      A(I,J,K) = A(I,J-1,K+1) + A(I-1,J,K+1)
    ENDDO
  ENDDO
ENDDO
```

0	+	+
+	0	+

Loop Reversal

- When the iteration space of a loop is reversed, the direction of dependences within that reversed iteration space are also reversed
  - A “+" dependence becomes a “-" dependence, and vice versa
- We cannot perform loop reversal if the loop carries a dependence

Perform Interchange after Loop Reversal

```
DO K = L, 1, -1
  DO I = 2, N+1
    DO J = 2, M+1
      A(I,J,K) = A(I,J-1,K+1) + A(I-1,J,K+1)
    ENDDO
  ENDDO
ENDDO
```

Increases options for performing other optimizations

Parallelize loops I and J

+	0	+
+	+	0

Which Transformations are Most Important?

- Selecting the best loops for parallelization is a NP-complete problem
- Flow dependences by nature are difficult to remove
  - Try to reorder statements as in loop peeling, loop distribution
- Techniques like scalar expansion, privatization can be useful
  - Loops often use scalars for temporary values

+	+	0	0
+	0	+	0
+	0	0	+
0	+	0	0
0	0	+	0
0	0	0	+

Carries most dependences!

Challenges for Real-World Compilers

- Conditional execution
- Symbolic loop bounds
- Indirect memory accesses
- …

References

- R. Allen and K. Kennedy – Optimizing Compilers for Multicore Architectures, Chapters 5-6.
- S. Midkiff – Automatic Parallelization: An Overview of Fundamental Compiler Techniques.
- P. Sadayappan and A. Sukumaran Rajam – CS 5441: Parallel Computing, Ohio State University.