Homework 2

November 8, 2023

1. Using the Cauchy-Schwarz inequality, establish that for every vector $v \in \mathbb{R}^n$, $|v|/\sqrt{n} \leq ||v||$. (5 points)

2. Prove that if G is an n-vertex bipartitte graph and A its adjacency matrix, then there is a vector $v \in \mathbb{R}^n$such that $Av = -v$. (10 points)

3. Let A, B be two symmetric stochastic matrices. Show that $\lambda(A + B) \leq \lambda(A) + \lambda(B)$. (10 points)

4. Extend the error-reduction procedure using random walks on expanders we discussed in the class, to the complexity class BPP. Recall that BPP algorithms have two-sided error. (15 points)