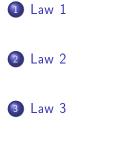
Monad Laws

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Monad Laws

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→ E → → E →

Image: A matrix

return $x \gg f = f x$

This means that a monad containing x when passed to f, should return the same value as f applied to (unboxed) x.



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 $mx \gg return = mx$ This is the dual of the first law. This says that a "boxed" x when passed to return should evaluate to the "boxed" x itself. One way to see this as follows: return unboxes x from x, and then evaluates to the boxed value x. This is in fact, mx itself.



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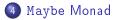
→ E → → E →

Image: A matrix

 $(mx \gg f) \gg g = mx \gg (\langle x - f x \rangle g)$ This is the associative law for bind. Note that any associative law involves only one operator.

We now verify these laws are satisfied by the Maybe monad





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instance Monad Maybe where return x = Just x

$$mx >>= f = f x$$

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Monadic Law 1 for Maybe

return
$$x \gg f$$

= (Just x) $\gg f$
= f x

which satisfies the Monadic Law 1

[definition of return] [definition of >>=]

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Monadic Law 2 for Maybe

[definition of >>=]

which satisfies the Monadic Law 2

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Law 3

Monadic Law 3 for Maybe

This satisfies the Monadic Law 3.

Law 3

Monadic Law 3 for Maybe

This satisfies the Monadic Law 3.

Law 3

Monadic Law 3 for Maybe

This satisfies the Monadic Law 3. (the derivation can be shortened, of course.)