

MID-SEMESTER EXAMINATION (2024-25/I)

POINTS: 60

DATE GIVEN: 14-SEP

DUE: 18-SEP-2024 (6PM)

Rules:

- You are not allowed to discuss.
- Write the solutions on your own and honorably *acknowledge* the sources if any. <http://cse.iitk.ac.in/pages/AntiCheatingPolicy.html>
- Submit your solutions, before time, to your Tutor (CS Bhargav). Preferably, give the Tutor a printed copy of your LaTeXed or Word processed solution sheet.
- Clearly express the fundamental *idea* of your proof/ algorithm before going into the other proof details. The distribution of partial marks is according to the proof steps.
- There will be a penalty if you write unnecessary or unrelated details in your solution. Also, do not repeat the proof details covered before.

Question 1: [10 points] Consider the polynomial $f = x_1x_2 \cdots x_{n/2} + x_{n/2+1}x_{n/2+2} \cdots x_n$ in the polynomial ring $\mathbb{F}_2[x_1, \dots, x_n]$, for even $n \geq 4$. Prove that $\forall e \in \mathbb{N}$, f cannot be written as a sum of e -th-powers of linear-polynomials.

Question 2: [15+5 points] Let us introduce a *division* gate in the arithmetic circuit model. Let $f \in \mathbb{C}[x_1, \dots, x_n]$ be a degree d polynomial computed by a circuit of size s , using $\{+, \times, \div\}$ gates. Show that f has a circuit of size $\text{poly}(sd)$ using only the $\{+, \times\}$ gates.

Does the above result work over any field?

The depth reduction proofs unfold several structural properties of arithmetic circuits. Two of them will be discussed below.

Question 3: [15 points] VNP was defined by taking projections of a circuit $C(\mathbf{x}, \mathbf{y})$, in $\mathbf{y} \in \{0, 1\}^n$. Analogously, define VNP_e by taking the sum of all projections of a *formula* $F(\mathbf{x}, \mathbf{y})$, in $\mathbf{y} \in \{0, 1\}^n$. Show that $\text{VNP} = \text{VNP}_e$.

Question 4: [5+10 points] A circuit C is called *weakly-skew* if each of its multiplication gate α has two inputs β and γ such that: at least one of the subcircuits C_β or C_γ is connected to the rest of the circuit only via α . The class of VP-polynomials computed by a poly-sized weakly-skew circuit is called VP_{ws} .

Show that $\text{VBP} = \text{VP}_{ws}$.

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