
CS748 - ARITHMETIC CIRCUIT COMPLEXITY
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END-SEMESTER EXAMINATION (2024-25/I)

POINTS: 80

DATE GIVEN: 20-NOV'24

DUE: 24-NOV (7PM)

Rules:

- You are not allowed to discuss.
- Write the solutions on your own and honorably *acknowledge* the sources if any. <http://cse.iitk.ac.in/pages/AntiCheatingPolicy.html>
- Email your solutions, before time, to your Tutor (CS Bhargav) and me. Preferably, send us a pdf of your LaTeXed or Word processed solution sheet. Otherwise, make sure that the scan is very clear and easy to read.
- Clearly express the fundamental *idea* of your proof/ algorithm before going into the other proof details. The distribution of partial marks is according to the proof steps.
- There will be a penalty if you write unnecessary or unrelated details in your solution. Also, do not repeat the proof details covered before.

Question 1: [Shallow ckts] [16 points] Suppose a degree d polynomial f has a VP circuit of size s . For any $\Delta > 0$, show that f has a product-depth- Δ circuit of size $s^{O(d^{1/\Delta})}$.

Question 2: [hsg→lb] [14 points] Show that if there is a poly(s)-time blackbox PIT algorithm for size s , degree s circuits, then either $E \not\subseteq \#P/\text{poly}$ OR there is a polynomial family $\{q_m\}_m$ in VNP that requires size $2^{\Omega(m)}$ VP circuits.

Question 3: [lb→hsg] [20+30 points] Recall *diagonal depth-4* circuits, which is an important model to study in VP. Consider

$$C = \sum_{i=1}^k c_i \cdot Q_i^{d_i} \in \mathbb{Q}[x_1, \dots, x_n],$$

where $c_i \in \mathbb{Q}$, $d_i \in \mathbb{N}$, and Q_i 's are polynomials of degree at most δ . In this problem we are interested in the setting when δ is quite *small* (even the case of $\delta = 2$ is interesting!).

i) Prove that any such C computing the monomial $x_1 \cdots x_n$ satisfies $k \geq 2^{\Omega(n/\delta)}$.

[Hint: Use shifted partials measure.]

ii) Let C be given as a blackbox (with the promised parameters and size s). Construct a hitting-set in time $\text{poly}(s^{\delta \cdot \log s})$.

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