

MID-SEMESTER EXAMINATION (2025-26/I)

POINTS: 40

DATE GIVEN: 16-SEP'25

DUE: 19-SEP'25 (EOD)

Rules:

- You are *not* allowed to discuss.
- Write the solutions on your own and honorably *acknowledge* the sources if any. <http://cse.iitk.ac.in/pages/AntiCheatingPolicy.html>
- Submit your solutions, before time, to your TA. Please give your LaTeXed or Word processed solution-sheet in PDF. This will be graded, and commented, in-place.
- Clearly express the fundamental *idea* of your proof/ algorithm before going into the other proof details. The distribution of partial marks is according to the proof steps.
- There will be a penalty if you write unnecessary or unrelated details in your solution. Also, do not repeat the proof details covered before.

Solve any two of Q.1-3, and solve Q.4. In all, you have to attempt three problems.

Question 1: [10 points] Show that $P \neq BPP$ implies $P \neq NP$.

Question 2: [10 points] Recall Razborov-Smolensky's approximator polynomial technique. Can it be generalized to prove $\text{mod}_p \notin \text{ACC}^0[q^m]$, where p, q are distinct primes and $m \in \mathbb{N}$? Give a detailed response.

Question 3: [10 points] Let G be an undirected non-complete regular graph with self-loops. Let A be its normalized adjacency matrix. Show that the eigenvalue of A with the second-largest *value* (note- we are not considering the absolute value) is positive.

Question 4: [15+5 points] Let G be an (n, d, λ) -expander. Prove the following combinatorial expansion property: For all $S, T \subseteq V(G)$,

$$\left| |E(S, T)| - \frac{d}{n} |S| \cdot |T| \right| \leq \lambda d \cdot \sqrt{|S| \cdot |T|}.$$

$E(S, T)$ denote the set of edges from S to T .

Deduce an upper bound on the size of *independent sets* in G .

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