CS747 - RANDOMIZED METHODS IN COMPUTATIONAL COMPLEXITY
NITIN SAXENA

MID-SEMESTER EXAMINATION (2023-24/I)

POINTS: 40

DATE GIVEN: 21-SEP'23 DUE: 25-SEP'23 (10AM)

Rules:
- You are *not* allowed to discuss.
- Write the solutions on your own and honorably *acknowledge* the sources if any. `http://cse.iitk.ac.in/pages/AntiCheatingPolicy.html`
- Submit your solutions, before time, to your TA. Please give your LaTeXed or Word processed solution-sheet in PDF. This will be graded, and commented, in-place.
- Clearly express the fundamental *idea* of your proof/ algorithm before going into the other proof details. The distribution of partial marks is according to the proof steps.
- There will be a penalty if you write unnecessary or unrelated details in your solution. Also, do not repeat the proof details covered before.

Solve any two of Q.1-3, and solve Q.4. In all, you have to attempt three problems.

Question 1: *[10 points]* Show that $\mathrm{P} \neq \mathrm{BPP}$ implies $\mathrm{P} \neq \mathrm{NP}$.

Question 2: *[10 points]* Recall Razborov-Smolensky's approximator polynomial technique. Can it be generalized to prove $\mathrm{mod}_p \notin \mathrm{ACC}^0[q^m]$, where p, q are distinct primes and $m \in \mathbb{N}$? Give a detailed response.

Question 3: *[10 points]* Let G be an undirected non-complete regular graph with self-loops. Let A be its normalized adjacency matrix. Show that the eigenvalue of A with the second-largest *value* (note- we are not considering the absolute value) is positive.

Question 4: *[15+5 points]* Let G be an (n, d, λ)-expander. Prove the following combinatorial expansion property: For all $S, T \subseteq V(G)$,

$$\left| |E(S,T)| \ - \ \frac{d}{n}|S| \cdot |T| \right| \ \leq \ \lambda d \cdot \sqrt{|S| \cdot |T|} \, .$$

$E(S, T)$ denote the set of edges from S to T.

Deduce an upper bound on the size of *independent sets* in G.

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