CS747 - RANDOMIZED METHODS IN COMPUTATIONAL COMPLEXITY

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ASSIGNMENT 2

POINTS: 50

DATE GIVEN: 31-AUG-2018 DUE: 14-SEP-2018

Rules:

- You are strongly encouraged to work *independently*. That is the best way to understand the subject.
- Write the solutions on your own and honorably *acknowledge* the sources if any. `http://cse.iitk.ac.in/pages/AntiCheatingPolicy.html`
- Submit your solutions, before time, to your TA. Preferably, give the TA a printed copy of your LaTeXed or Word processed solution sheet.
- Clearly express the fundamental *idea* of your proof/ algorithm before going into the other proof details. The distribution of partial marks is according to the proof steps.
- There will be a penalty if you write unnecessary or unrelated details in your solution. Also, do not repeat the proofs done in the class.
- Problems marked '0 points' are for practice.
- Acknowledgements: Several problems are from *Arora & Barak, Computational Complexity: A Modern Approach* and other lecture notes.

Question 1: *[6+3 points]* In the class we proved the Razborov-Smolensky result: $\mathrm{mod}_p \notin \mathrm{ACC}^0[q]$, for $p = 2 = q - 1$. Give the general proof for primes $p \neq q$.

What happens to the proof when one of them is a composite?

Question 2: *[7 points]* Show that the OR of n variables cannot be expressed as a polynomial over $\mathbb{F}_p$ of degree less than n.

Question 3: *[6+9 points]* Let A be a real symmetric stochastic matrix. Show that

(1) its eigenvalues are real with absolute value at most 1.
(2) its eigenvectors, for different eigenvalues, are linearly independent.

Question 4: *[6 points]* Let A, B be real symmetric stochastic matrices. Show that the *second-largest eigenvalue* function $\lambda(\cdot)$ is *subadditive* (i.e. $\lambda(A+B) \leq \lambda(A) + \lambda(B)$).

Question 5: *[13 points]* Let G be a graph with only *two* distinct eigenvalues. Prove that it is complete.

Question 6: *[0 points]* Show that for an $n \times n$ matrix $X = ((x_{i,j}))$, the polynomial $\text{per}(X)$ has a formula of size $2^{O(n)}$.

Question 7: *[0 points]* Show that $\text{AM}[k] = \text{AM}[2]$ for all constant $k \geq 2$.

Question 8: *[0 points]* Show that the graph isomorphism problem is in NP∩coAM. (So, "almost" in NP∩coNP.)

Question 9: *[0 points]* Fix parameters n, d. Show that a random n-vertex d-regular graph is a $(n, 2d, 1/3)$-combinatorial expander.

Question 10: *[0 points]* Consider a subset A of the n-tuples $\mathbb{F}_3^n$. We are interested in the largest set A that is *free of arithmetic progressions of size* 3 (i.e. there is no $a, b, c \in A$ satisfying $a + b + c \equiv \mathbf{0} \mod 3$).

Could you relate this question to the sunflower lemma?

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