a directory of all known zeta functions

[This page is under continual construction! Any contributions would be welcome.]

"*Over the years striking analogies have been observed between the Riemann zeta-function and other zeta- or L-functions. While these functions are seemingly independent of each other, there is growing evidence that they are all somehow connected in a way that we do not fully understand. In any event, trying to understand, or at least classify, all of the objects which we believe satisfy the* [Riemann hypothesis](#) *is a reasonable thing to do.*"

J. Brian Conrey, "The Riemann Hypothesis", *Notices of the AMS* (March, 2003) p.347

"*In this essay I will give a strictly subjective selection of different types of zeta functions. Instead of providing a complete list, I will rather try to give the central concepts and ideas underlying the theory...*

Whenever entities are counted with some mathematical structure on them it is likely that a zeta function can be set up and often enough it will extend to a meromorphic function. Zeta functions show up in all areas of mathematics and they encode properties of the counted objects which are well hidden and hard to come by otherwise. They easily give fuel for bold new conjectures and thus drive on mathematical research. It is a fairly safe assertion to say that zeta functions of various kinds will stay in the focus of mathematical attention for times to come. "

A. Deitmar, "Panorama of zeta functions" (preprint 10/02)

"Some decades ago I made - somewhat in jest - the suggestion that one should get accepted a non-proliferation treaty of zeta functions. There was becoming such an overwhelming variety of these objects."

Atle Selberg, quoted in K. Sabbagh, *Dr. Riemann's Zeros* (Atlantic, 2002)

"*In concentrating exclusively on the study of the* [Riemann] *zeta function and its relation to the prime number theorem, this book ignores one of the most fruitful areas of development of Riemann's work, namely, number theory. The use of functions like the zeta function in number theory was a major feature of the work of Dirichlet- both in his* L*-series and in his formula for the class number of a quadratic number field - many years before Riemann's paper appeared, and the use of such functions has been a prominent theme in number theory ever since. Riemann's contributions in this area were primarily function-theoretic, not number-theoretic, and consisted of focussing attention on the functions as functions of a* complex *variable, on the possibility of their satisfying a functional equation under* s <--> 1 - s*, and on the importance of the location of their complex zeros. A few of the most important names in the subsequent study of these number theoretic functions are those of Dedekind, Hilbert, Hecke, Artin, Weil, and Tate.*

Ignorance prevents me from entering into a discussion of these functions and what is known about them. However, it seems that they provide some of the best reasons for believing that the Riemann hypothesis is true - for believing, in other words, that there is a profound and as yet uncomprehended number-theoretic phenomenon, one facet of which is that the roots all lie on Re[s] = 1/2. In particular, there is a 'zeta function' associated in a natural number-theoretic way to any function field over a finite field, and Weil has shown that the analog of the Riemann hypothesis is true for such 'zeta functions'"

H.M. Edwards, from *Riemann's Zeta Function* (Academic Press, 1974) p. 298

Some useful discussion of the very fundamental question "*what is a zeta function?*" took place at the 2002 conference on zeta functions and associated Riemann Hypotheses in New York.

As M. Huxley puts it: "*We know one when we see one.*"

J. Baez's *This Weeks Finds in Mathematical Physics* Week 216 contains some useful discussion about various types of zeta functions. week 217 includes very helpful discussion of the Riemann Hypothesis, Extended Riemann Hypothesis, Grand Riemann Hypothesis, Weil Conjectures, Langlands Programme, the functional equations of zeta and *L*-functions, modularity of theta functions, *etc.* week 218 follows this up, framing certain issues concerning the bewildering array of zeta and *L*-functions in terms of category theory.

P.E. Cartier, B. Julia, P. Moussa and P. Vanhove (eds.), *Frontiers in Number Theory, Physics, and Geometry: On Random Matrices, Zeta Functions, and Dynamical Systems* (Springer, 2006)

Probably the best overall view of zeta functions as a general phenomenon appears in the Mathematical Society of Japan's *Encyclopedic Dictionary of Mathematics* (pp. 1372-1392 of the English edition published by MIT Press in 1977)

The introductory section informs us:

"Since the 19th century, many special functions called zeta functions have been defined and investigated. The four main problems concerning zeta functions are:

(1) Creation of new zeta functions.

(2) Investigation of the properties of zeta functions. Generally, zeta functions have the following four properties in common: (i) They are meromorphic on the whole complex plane; (ii) they have Dirichlet series expansions; (iii) they have Euler products expansions; and (iv) they satisfy certain functional equations. Also it is an important problem to find the poles, residues, and zeros of zeta functions.

(3) Application to number theory, in particular to the theory of decomposition of prime ideals in finite extensions of algebraic number fields. (4) Study of the relations between different zeta functions.

Most of the functions called zeta-functions or *L*-functions have the four properties of problem (2). The following is a classification of the important types of zeta functions that are already known, which will be discussed later in this article:

(1) The zeta and *L*-functions of algebraic number fields: the Riemann zeta function, Dirichlet *L*-functions (study of these functions gave impetus to the theory of zeta functions), Dedekind zeta functions, Hecke *L*-functions, Hecke *L*-functions with Grossencharakters, Artin *L*-functions, and Weil *L*-functions.

(2) The *p*-adic *L*-functions related to the works of H.W. Leopoldt, T. Kubota, K. Iwasaw, etc.

(3) The zeta functions of quadratic forms: Epstein zeta functions, zeta functions of indefinite quadratic forms (C.L. Siegel), etc.

(4) The zeta and *L*-functions of simple algebras: Hey zeta functions and the zeta functions given by R. Godement, T. Tamagawa, etc.

(5) The zeta functions associated with Hecke operators, related to the work of E. Hecke, M. Eichler, G. Shimura, etc.

(6) The congruence zeta and *L*-functions attached to algebraic varietites defined over finite fields (E. Artin, F.K. Schmidt, A. Weil); Hasse zeta functions attached to the algebraic varietites defined over algebraic number fields.

(7) The zeta functions attached to discontinuous groups: Selberg zeta functions, the Eisenstein series defined by A. Selberg, Godement, and I.M. Gel'fand, etc.

(8) Y. Ihara's zeta function related to non-Abelian class field theory over a function field over a finite field.

(9) zeta functions associated with prehomogeneous vector spaces."

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alphabetic directory

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zeta function of an **Abelian variety**

G. Shimura, *Abelian Varieties with Complex Multiplication and Modular Functions* (Princeton, 1997)

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zeta function of an **algebraic curve over a finite field**

M. Deurling, "The zeta-functions of algebraic curves and varieties", *Report of an International Colloquium on Zeta-Functions* (1956), K. Chandrasekharan, editor.

W. A. Zúñiga-Galindo, "Zeta functions of singular curves over finite fields", *Revista Colombiana de Matematicas* **31** (1997) 115-124.

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zeta functions for **Anosov flows**

D. Ruelle, "Zeta functions for expanding maps and Anosov flows", *Inventiones Math.* **34** (1976) 231-242.

Artin-Mazur zeta function

D. Ruelle, "Dynamical zeta functions and transfer operators"

Ruelle explains that Artin-Mazur zeta functions are Weil zeta functions in the case where we have a diffeomorphism on a compact manifold.

D. Ruelle, "Zeta functions and statistical mechanics", *Asterisque* **40** (1976), 167-176.

Artin-Weil zeta function

DFG research group - "Transfer operators and dynamical zeta functions"

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zeta function of an **attractor**

R. Williams, "The zeta function of an attractor", *Conference on the Topology of Manifolds*, (editors J.C. Hocking, *et. al.*) (1968), 155-161.

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zeta functions of **automorphisms of free groups**

M. Lustig, *et. al.*, Geodesics on flat surfaces with conical singularities/ zeta function for automorphisms of free groups

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Barnes zeta function

J.S. Dowker and K. Kirsten, "The Barnes zeta-function, sphere determinants and Glaisher- Kinkelin-Bendersky constants"

Beurling modified zeta functions **for systems of *g*-primes**

E. Stankus, "Modified zeta functions and the number of *g*-integers"

Beurling *g*-primes bibliography and notes

Burgess zeta functions

"The main online reference (also mentioned in [2]) appears to be the 2009 paper by Terry Tao: A remark on partial sums involving the Moebius function [3], in which he defines them as analagous to the Riemann zeta function, but acting only over the multiplicative semigroup generated by a given set of primes $\mathcal{P}$: $\zeta_\mathcal{P}$ is then defined for $Re(s) > 1$ by the formula $\zeta_\mathcal{P}(s) := \sum_{n \is <\mathcal{P}>}{\frac{1}{n^s}} = \prod_{p \is \mathcal{P}}{ (1 - \frac{1}{p^s})^{-1}}$

[2] http://mathoverflow.net/questions/28000/what-are-the-analytic-properties-of-dirichlet-euler-products-restricted-to-arithm
[3] http://arxiv1.library.cornell.edu/pdf/0908.4323

[Hugo van der Sanden]

Carlitz zeta function

J.-P. Allouche, "Finite automata and arithmetic"

G. Damamme and Y. Hellegouarch, "Transcendence of the values of the Carlitz zeta function by Wade's method", *Journal of Number Theory* **39** (1991) 257-278.

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zeta functions for **crystallographic groups**

M. du Sautoy, J. McDermott, and G. Smith, "Zeta functions of crystallographic groups and analytic continuation", *Proceedings of the London Mathematical Society* **79** (3) (1999) 511-534.

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zeta functions associated with **curves over finite fields**

Artin conjectured that the Riemann hypothesis holds for the zeta function associated with any curve over a finite field. Hasse proved this for elliptic curves. Andre Weil (while in prison during WWII) proved it for arbitrary curves. Deligne went on to prove it for general algebraic varieties.

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zeta functions of **cyclotomic fields**

http://www.princeton.edu/~missouri/Generals/generals/prasanna_kartik

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Dedekind zeta functions

WWN notes on Dedekind zeta functions (part of a work-in-progress)

From J.T. Tate, "Fourier analysis in number fields and Hecke's zeta-functions" [1950 Princeton Ph.D. thesis, reproduced as Chapter 15 of *Algebraic Number Theory* by J.W.S. Cassels and A. Fröhlich (Academic Press, 1967)]:

"Hecke was the first to prove that the Dedekind zeta-function of *any* algebraic number field has an analytic continuation over the whole plane and satisfies a simple functional equation. He soon realized that this method would work, not only for the Dedekind zeta-function and L-series, but also for a zeta-function formed with a new type of ideal character which, for principal ideals depends not only on the residue class of the number modulo the "conductor", but also on the position of the conjugates of the number in the complex field. Overcoming rather extraordinary technical complications, he showed (1918 and 1920) that these "Hecke" zeta-functions satisfied the same type of functional equation as the Dedekind zeta-function, but with a much more complicated factor."

P. Cohen, Dedekind zeta functions and quantum statistical mechanics

[abstract] "In this article we construct a C*-dynamical system with partition function the Dedekind zeta function of a given number field and with a phase transition at the pole of this zeta function which detects a breaking of symmetry with respect to a natural symmetry group. This extends work of Bost-Connes and Harari-Leichtnam."

http://at.yorku.ca/cgi-bin/amca/cadx-04

See also Appendix A.1 of *Fractal Geometry and Number Theory* by M.L. Lapidus and M. van Frankenhuysen

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zeta function of a **discrete group of automorphisms of a bounded degree tree**

http://euler.slu.edu/Dept/Faculty/clair/papers.html

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zeta function of a **distributive lattice**

K.H. Knuth, "Lattice duality: The origin of probability and entropy", *Neurocomputing* **67** C (2005) 245-274

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zeta function of a **division algebra**

T. Tamagawa, "On the zeta function of a division algebra", *Annals of Mathematics* **77** (1963) 387-405.

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dynamical zeta functions

number theory and physics archive page

A. Juhl, *Cohomological Theory of Dynamical Zeta Functions* (Progress in Mathematics, Vol. 194.) (Birkhauser, 2001)

R. Mainieri, Arithmetical properties of dynamical zeta functions

http://www.math.psu.edu/dynsys/semSpring00.html

http://www.geom.umn.edu/~rminer/talks/cecm/ttmath/Ruelle2.html

http://www.ma.man.ac.uk/~mp/research.html

Elizalde zeta functions

A class of zeta functions that extends the class of Epstein's was recently brought to my attention by Prof. E. Elizalde of M.I.T. Although I don't think they've appeared in print under this name, it seems an appropriate one to give them. They are spectral zeta functions associated with a quadratic + linear + constant form in any number of dimensions. Elizalde has developed formulas for them which extend the famous Chowla-Selberg formula.

E. Elizalde, "Explicit zeta functions for bosonic and fermionic fields on a noncommutative toroidal spacetime", *Journal of Physics A* **34** (2001) 3025-3036.

E. Elizalde, "Multidimensional extension of the generalized Chowla-Selberg formula", *Communications in Mathematical Physics* **198** 91998) 83-95.

E. Elizalde, "Zeta functions, formulas and applications", *J. Comp. Appl. Math.* **118** (2000) 125.

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zeta functions of **elliptic operators**

S. Moronianu, "Adiabatic limits of eta and zeta functions of elliptic operators"

http://www.math.ohio-state.edu/Graduate/THESIS_ABSTRACTS/BUCICOVSCHI.BOGDAN.html

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zeta functions of **energy of PT-symmetric quantum systems**

http://www.physics.wustl.edu/graduate/archive/Wang111000.html

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Epstein zeta function

Heilbronn proved that the Riemann Hypothesis fails for the Epstein zeta function.

Eric Weisstein's notes

S. Chowla and A. Selberg, "On Epstein's zeta-function", *J. Reine und angew. math.* **227** (1967) 86-110.

D. Hejhal, "Zeros of Epstein zeta-functions and supercomputers", *Proc. Intern. Congress. Math.* (Berkeley, 1986) Vol. II (AMS, 1987) 1362-1384.

C. Siegel, "A generalization of the Epstein zeta function", *Report of an International Colloquium on Zeta-Functions* (1956), K. Chandrasekharan, editor.

U. Christian, *Selberg's Zeta-, L- and Eisensteinseries* (Lecture Notes in Mathematics **1030**, Springer, 1983)

http://www.aurora.edu/~ldelacey/vita2.htm

Appendix A.4 of *Fractal Geometry and Number Theory* by M.L. Lapidus and M. van Frankenhuysen

M.L. Glasser and I.J. Zucker, "Lattice Sums in Theoretical Chemistry." In *Theoretical Chemistry: Advances and Perspectives*, Vol. 5 (Ed. H. Eyring). New York: Academic Press, pp. 69-70, 1980.

D. Shanks, "Calculation and Applications of Epstein Zeta Functions." *Math. Comput.* **29** (1975) 271-287.

Estermann zeta function

http://www.fsci.fuk.kindai.ac.jp/~kanemitu/number.html

http://www.mscs.dal.ca/~dilcher/berni.html

Euler zeta function

as defined in the *Prime Pages* glossary

K. Devlin, "How Euler discovered the zeta function" (elementary historical introduction)

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zeta functions for **expanding maps**

D. Ruelle, "Zeta functions for expanding maps and Anosov flows", *Inventiones Math.* **34** (1976) 231-242.

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zeta function associated with **finite extension of the rational numbers**

http://www.cs.bgu.ac.il/~saarh/colloquium/goren/goren.html

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zeta functions for **flows**

D. Ruelle, "Zeta functions and statistical mechanics", *Asterisque* **40** (1976), 167-176.

A. Juhl, *Cohomological Theory of Dynamical Zeta Functions* (Progress in Mathematics, Vol. 194.) (Birkhauser, 2001)

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zeta functions for **forms of Fermat equations**

L. Brünjes, *Forms of Fermat Equations and their Zeta Functions* (World Scientific, 2004)

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zeta function of a **generalised cone**

http://www-phys.science.unitn.it/research/consuntivi/ft97-campi.html

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geometric zeta functions

http://www.best.com/~worktree/g/87/243g.htm

publications of M.L. Lapidus

A. Deitmar, "Geometric zeta-functions on *p*-adic groups"

A. Deitmar, "Geometric zeta-functions of locally symmetric spaces", *Am. J. Math.* **122** vol.5 (2000) 887-926.

A. Deitmar, "Geometric zeta-functions, L^2-theory, and compact Shimura manifolds"

Goss zeta function

http://www.math.uiuc.edu/Algebraic-Number-Theory/0096/

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zeta function of a finite **graph**

A. Terras, *Zeta Functions of Graphs: A Stroll Through the Garden* (Cambridge Univ. Press, 2010)

A. Terras and H. Stark, "Zeta functions of finite graphs and coverings", *Advances in Mathematics* **121** (1996) 124-165.

A. Terras and H. Stark, "Zeta functions of finite graphs and coverings, Part II", *Advances in Mathematics*, **154** (2000), 132-195.

http://www.math.dartmouth.edu/~colloq/s97/stark.html

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zeta function of a finite unoriented **graph**

see **Ihara-Selberg** zeta function

Gutzwiller-Voros zeta function

http://www.mpg.de/reports/9814/9814_T.htm

http://130.83.24.4/nhc/activities/ChaoticScattering.html

Hasse-Weil zeta function **(of an elliptic curve)**

http://www.dpmms.cam.ac.uk/Algebraic-Number-Theory/0095/

http://www.math.purdue.edu/research/seminars/old_abstracts/1999/ html/abs_11_30_99a.html

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Hawking zeta function

http://www.oa.uj.edu.pl/~maslanka/

http://citeseer.nj.nec.com/114922.html

http://citeseer.nj.nec.com/smith95fundamental.html

Hecke zeta functions

"Hecke was the first to prove that the Dedekind zeta-function of *any* algebraic number field has an analytic continuation over the whole plane and satisfies a simple functional equation. He soon realized that this method would work, not only for the Dedekind zeta-function and L-series, but also for a zeta-function formed with a new type of ideal character which, for principal ideals depends not only on the residue class of the number modulo the "conductor", but also on the position of the conjugates of the number in the complex field. Overcoming rather extraordinary technical complications, he showed (1918 and 1920) that these "Hecke" zeta-functions satisfied the same type of functional equation as the Dedekind zeta-function, but with a much more complicated factor."

From J.T. Tate, "Fourier analysis in number fields and Hecke's zeta-functions" [1950 Princeton Ph.D. thesis, reproduced as Chapter 15 of *Algebraic Number Theory* by J.W.S. Cassels and A. Fröhlich (Academic Press, 1967)]

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height zeta functions

J. Shalika and Y. Tschinkel, "Height zeta functions of equivariant compactifications of the Heisenberg group"

Hey zeta functions

P. Roquette, "Class field theory in characteristic *p*, its origin and development"

Hlawka zeta function

E. Hlawka, "Uber die Zetafunktion konvexer Körper", *Monatsh. Math.* **54** (1950) 81-99.

The term "Hlawka's zeta-function" has recently used by (among others) Martin Huxley.

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zeta function of a **homeomorphism**

http://www.math.nwu.edu/graduate/prelims/dyna88.pdf

Hurwitz zeta function

J. Borwein, D. Bradley and R. Crandall, "Computational strategies for the Riemann zeta function", *J. Comp. App. Math.* **121** (2000) p.8

defined as a generalisation of Riemann's zeta function

Eric Weisstein's notes

V. Adamchik, "Derivatives of the Hurwitz zeta function for rational arguments", *Journal of Computational and Applied Mathematics* **100** (1999) 201-206.

A. Veselov and J. Ward, "On the real roots of the Bernoulli polynomials and the Hurwitz zeta-function" (1999)

J. Andersson, "Mean value properties of the Hurwitz zeta-function", *Mathematica Scandinavica* **71** (1992) 295-300.

V. Adesi and S. Zerbini, "Anayltic continuation of the Hurwitz zeta function with physical applications"

O. Espinosa and V. Moll, "On some integrals involving the Hurwitz zeta function: part 2"

M. Katsurada and K. Matsumuto, "Explicit formulas and asymptotic expansions for certain mean square of Hurwitz zeta-functions", *Proc. Japan Acad.* **69** (8) (1993) 303-307.

V.V. Rane, "On Hurwitz zeta-function", *Math. Ann.* **264** (2) 147-151.

W.P. Zhang, "On the mean square value of the Hurwitz zeta-function", *Illinois Journal of Mathematics* **38** (1) (1994) 71-78.

Igusa local zeta function

http://www.mtholyoke.edu/~robinson/reu/reu95/reu95.html

POLYGUSA - Computer program to calculate Igusa's local zeta function associated to a polynomial

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Ihara-Hashimoto-Bass zeta function

http://euler.slu.edu/Dept/Faculty/clair/clair.html

Ihara-Selberg zeta function (zeta function of a finite unoriented graph)

D. Ruelle, "Dynamical zeta functions and transfer operators", p.4

Ruelle explains that they are defined in terms of the Euler product formula for Weil zeta functions, where periodic orbits are replaced by cycles (circuits on the graph in question without immediate backtracking). The reciprocals of these zeta functions are known to be polynomials, and the functions themselves are known to satisfy Riemann hypotheses precisely when the graph in question is Ramanujan.

http://www.pdmi.ras.ru/preprint/2000/00-07.html

H. Bass, "The Ihara-Selberg zeta function of a tree lattice", *Int. J. Math.***3** No. 6 (1992) 717-797.

Incomplete (Riemann) zeta function

K.S. Kolbig,"Complex zeros of an incomplete Riemann zeta function and of the incomplete gamma function", *Math. Comput.* **24** (1970) 679-696.

[**Jacobi** zeta function]

M. Somos points out "*...not at all like the other zeta functions mentioned. It is an elliptic function with double quasi-periodicity. There is nothing like the non-trivial zeros a la Riemann Hypothesis...note that it is just a historical accident that it was called a zeta function and has nothing to do with the rest...As a curiousity, the Jacobi theta function is involved with the functional equation of the Riemann zeta function via the Mellin transform. However, this is as close as it gets regarding Jacobi and his elliptic functions.*"

Köhler zeta functions

http://www.mathematik.uni-leipzig.de/GK/GKKolloquium.html

R. Berndt, "Köhler's computation of his Zeta function for an arithmetic curve of degree two", *Mitt. Math. Ges. Hamburg III* (Hamburg, 1985)

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zeta functions associated with **Laplace-type operators**

http://www.na.infn.it/gravity2001/qgprogram.htm

Kurokawa multiple zeta functions

N. Kurokawa, "Multiple zeta functions: an example", *Adv. Studies in Pure Math., Zeta functions in geometry* (1991)

Lefschetz zeta function

D. Ruelle, "Dynamical zeta functions and transfer operators"

Ruelle defines the Lefschetz zeta function analogously to the Weil zeta function, except fixed points are weighted by their topological indices. He points out that in many interesting cases all topological indices equal 1, in which case the Lefschetz zeta function becomes identical to the Weil zeta function.

Lerch zeta function

http://www.mif.vu.lt/~garunkstis

J. Borwein, D. Bradley and R. Crandall, "Computational strategies for the Riemann zeta function", *J. Comp. App. Math.* **121** (2000) p.11

R. Garunkstis and A. Laurincikas, "The Lerch zeta-function", *Integral Transforms and Special Functions* **10** (3-4) (2000) 211-226

J. Ignataviciute, "A limit theorem for the Lerch zeta-function", Special issue of *Lietuvos Matematikos Rinkinys* **40** (2000): *Proceedings of XLI Conference of Lithuanian Mathematical Society*, Ŝiauliai, June 22-23, 2000, 21-27

A. Laurincikas, "On the mean square of the Lerch zeta-function with respect to the parameter", *Proceedings of XLI Conference of Lithuanian Mathematical Society*, Ŝiauliai, June 22-23, 2000, 43-48

A. Laurincikas and K. Matsumoto, "The joint universality and the functional independence for Lerch zeta-functions", *Nagoya Journal of Mathematics* **157** (2000) 211-227

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zeta function of **Lyapunov exponent of a product of random matrices**

http://ups.cs.odu.edu/buckets/ups.xxx.chao-dyn/xxx.xxx.chao-dyn.9301001/

Matsumoto zeta function

R. Kacinskaite, "A discrete limit theorem for the Matsumoto zeta-function on the complex plane", *Lietuvos Matematikos Rinkinys* **40** (4) (2000) 475-492, (in Russian) *Lithuanian Mathematical Journal* 40(4) (2000) 364-378.

R. Kacinskaite, "On the value distribution of Matsumoto zeta-function on the complex plane", Special issue of *Lietuvos Matematikos Rinkinys* **40** (2000): *Proceedings of XLI Conference of Lithuanian Mathematical Society*, Ŝiauliai, June 22-23, 2000, 33-38.

Minakshisundaram-Pleijel zeta function

http://mmf.ruc.dk/~Booss/recoll.pdf

S. Minakshisundaram and A. Pleijel, "Some properties of the eigenfunctions of the Lapalace operator on Riemannian manifolds", *Canadian Journal of Mathematics* **1** (1949) 242-256.

H.P. McKean, "Selberg's trace formula as applied to a compact Riemann surface", *Communications on Pure and Applied Mathematics* **25** (1972) 225-246.

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motivic zeta function

http://www.wis.kuleuven.ac.be/wis/algebra/NotesCambridge/Naive%20motivic%20zeta%20function.htm

http://cwisdb.cc.kuleuven.ac.be/research/P/3E98/project3E980397.htm

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multiple-sum zeta functions

E. Elizalde, "Multiple zeta functions with arbitrary exponents", *Journal of Physics* A **22** (1989) 931-942.

Nielsen zeta function

http://www.yurinsha.com/317/ws11.1.htm

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zeta function associated with **nilpotent group** http://muse.jhu.edu/demo/ajm/

Nint zeta function

Eric Weisstein's notes

J.M. Borwein, *et.al.*, "Nearest Integer Zeta-Functions" *Amer. Math. Monthly* **101** (1994) 579-580.

Non-Abelian zeta functions

L. Weng, "Constructions of Non-Abelian Zeta Functions for Curves"

L. Weng, "Refined Brill-Noether Locus and Non-Abelian Zeta Functions for Elliptic Curves"

L. Weng, "Riemann-Roch, Stability and New Non-Abelian Zeta Functions for Number Fields"

L. Weng, " New Non-Abelian Zeta Functions for Curves over Finite Fields"

***p*-adic** zeta function

http://www-fourier.ujf-grenoble.fr/AIF/Vol38/E383_1/E383_1.html

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partial zeta functions

D. Wan, "Partial zeta functions of algebraic varieties over finite fields"

J.-P. Jurzak, "Partial Euler products as a new approach to Riemann Hypothesis"

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periodic zeta function

Eric Weisstein's notes

A. Kacenas and A. Laurincikas, "A note on the value-distribution of the periodic zeta-function", Special issue of *Lietuvos Matematikos Rinkinys* **40** (2000): *Proceedings of XLI Conference of Lithuanian Mathematical Society*, Ŝiauliai, June 22-23, 2000, 28-32.

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zeta function of **Picard modular surfaces**

http://www.math.ias.edu/~goresky/publ.html

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zeta functions for **piecewise monotonic transformations**

http://www.math.chs.nihon-u.ac.jp/~mori/lectures.html

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zeta functions related to **poly-Bernoulli numbers** http://www.math.kindai.ac.jp/math/ohno/ohnore.html

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zeta function of certain **prehomogeneous vector spaces**

http://ups.cs.odu.edu/buckets/ups.xxx.math/xxx.xxx.math.9408212/

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prime zeta function

Eric Weisstein's notes

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probablilistic generalisation of the Riemann zeta function

N. Boston, "A probabilistic generalization of the Riemann zeta function", *Analytic Number Theory, Vol. 1*, Progr. Math. **138**, (Birkhauser, 1996) 155-162.

***q*-analogues** of the Riemann zeta function

I. Cherednik, "On *q*-analogues of Riemann's *zeta*"

M. Kaneko, N. Kurokawa, and M. Wakayama, "A variation of Euler's approach to values of the Riemann zeta function"

[abstract:] "An elementary method of computing the values at negative integers of the Riemann zeta function is presented. The principal ingredient is a new *q*-analogue of the Riemann zeta function. We show that for any argument other than 1 the classical limit of this *q*-analogue exists and equals the value of the Riemann zeta."

Redei zeta function

J.P.S. Kung, M. Ram Murty, G.-C. Rota, "On the Redei zeta function", *J. Number Theory* **12** (1980) 421-436

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zeta function of a **regular language**

http://theory.lcs.mit.edu/~dmjones/hbp/tcs/Authors/honkalajuha.html

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Reidemeister zeta function

http://www.yurinsha.com/317/ws11.1.htm

Riemann zeta function ("the grandmother of all zeta functions" - D. Ruelle)

number theory and physics archive page

Ruelle zeta function

S.J. Patterson, "On Ruelle's zeta function", *Israel Math. Conf. Proc.* **3** (1990) 163-184.

A. Juhl, *Cohomological Theory of Dynamical Zeta Functions* (Progress in Mathematics, Vol. 194.) (Birkhauser, 2001)

dynamical and spectral zeta functions archive page

http://www.nbi.dk/CATS/c_e_borel/steiner_course

http://www.geom.umn.edu/~rminer/talks/cecm/ttmath/Ruelle2.html

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Selberg zeta function

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Eric Weisstein's notes

A. Voros, "Spectral functions and the Selberg zeta function", *Communications in Mathematical Physics* **110** (1987) 439-465.

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U. Christian, *Selberg's Zeta-, L- and Eisensteinseries* (Lecture Notes in Mathematics **1030**, Springer, 1983)

In this book, the author proves the analytic continuation and functional equation for the Selberg zeta function.

Iwaniec, H., *Introduction to the Spectral Theory of Automorphic Forms*, 2^{nd} edition, Graduate Studies in Mathematics **53** (AMS, 2002)

Chapter 10 covers the Trace Formula, and on p.154 we find a helpful note about Selberg zeta functions:

"If you will, the Selberg zeta-function satisfies an analogue of the Riemann hypothesis. However, the analogy with the Riemann zeta-function is superficial. First of all, the Selberg zeta function has no natural development into Dirichlet series. Furthermore, the functional equation...resists any decent interpretation as a kind of Poisson summation principle. Nevertheless, modern studies of $Z(s)$ have caused a lot of excitement in mathematical physics (see [Sa1]). At least, one may say that the dream of Hilbert and Pólya of connecting the zeros of a zeta-function with eigenvalues of a self-adjoint operator is a reality in the context of $Z(s)$."

[Sa1] P. Sarnak, "Determinants of Laplacians", *Communications in Mathematical Physics* **110** (1987) 113-120.

http://www.wiley-vch.de/books/tis/eng/3-527-40072-9.html

http://www.nbi.dk/CATS/c_e_borel/steiner_course

http://www.isibang.ac.in/Smubang/as/publi.htm

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semi-simple zeta function of **quaternionic Shimura varieties**

http://206.67.72.201/catalog/np/may97np/DATA/3-540-62645-x.html

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Shintani zeta function

A. Yukie, *Shintani Zeta Functions* (LMS Lecture Note Series **183**, Cambridge University Press, 1993)

"The purpose of this book is to introduce an approach based on geometric invariant theory to the global theory of zeta functions for prehomogeneous vector spaces."

http://www.math.okstate.edu/preprint/1995.html

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zeta function of a **simplicial complex**

http://citeseer.nj.nec.com/orner96subspace.html

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zeta function of **singular curve over a finite field**

http://www.emis.de/journals/RCM/vol31-2/97310206.html

Solomon's zeta function

http://www.wits.ac.za/science/number_theory/jplkpub.htm

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spectral zeta function

http://www.maths.ex.ac.uk/~mwatkins/zeta/physics3.htm

http://journals.wspc.com.sg/mpla/preserved-docs/132/gon.pdf

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stochastic zeta function **(of a shift)**

http://www.math.washington.edu/~lind/Papers/spantree.pdf

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zeta function of a **stochastic matrix**

notes from James Propp

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R. Mainieri, Arithmetical properties of dynamical zeta functions

M. Holthaus and E. Kalinowski, "Condensate fluctations in trapped Bose gases: Canonical vs. microcanonical ensemble", *Annals of Physics* **270** (1998) 198-230.

M. Holthaus, K.T. Kapale, V.V. Kocharovsky and M.O. Scully, "Master equation vs. partition function: canonical statistics of ideal Bose-Einstein condensates", *Physica A* **300** (2001) 433-467.

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http://www.wis.kuleuven.ac.be/wis/algebra/NotesCambridge/Topological%20zeta%20function.htm

W. Veys, "Determination of the poles of the topological zeta function for curves", *Manuscripta Math.* **87** (1995), 435-448

W. Veys, "The topological zeta function associated to a function on a normal surface germ", *Topology* **38** (1999) 439-456

D. Segers and W. Veys, "On the smallest poles of topological zeta functions", *Compositio Math.* **140** (2004) 130-144

D. Segers, "Smallest poles of Igusa's and topological zeta functions and solutions of polynomial congruences"

A. Lemahieu, D. Segers and W. Veys, "On the poles of topological zeta functions", preprint (2004), 11pp.

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C.S. Calude and M. Stay, "Natural halting probabilities, partial randomness, and zeta functions" (preprint 01/06)

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two-variable zeta function for **number fields**

J.C. Lagarias, "On a two-variable zeta function for number fields"

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van der Geer-Schoof zeta function

http://at.yorku.ca/cgi-bin/amca/cadx-67

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M. Deurling, "The zeta-functions of algebraic curves and varieties", *Report of an International Colloquium on Zeta-Functions* (1956), K. Chandrasekharan, editor.

B. Dwork, "On the rationality of the zeta function of an algebraic variety", *American Journal of Mathematics* **82** (1960) 632-648.

See also **Weil** zeta functions

http://www.math.berkeley.edu/~ribet/Colloquium/dwan.html

G. Shimura, *Abelian Varieties with Complex Multiplication and Modular Functions* (Princeton, 1997)

[**Weierstrass** zeta function]

M. Somos points out "*...not at all like the other zeta functions mentioned...just a variant of the Jacobi zeta function.*"

Weil zeta functions

D. Ruelle, "Dynamical zeta functions and transfer operators"

Ruelle defines the Weil zeta function for an algebraic variety over a finite field in terms of the numbers of fixed points of all iterations of the Frobenius map on the extension of the algebraic variety to the algebraic closure of the finite field. He goes on to explain how the concept can be extended to more general maps on more general spaces. Weil zeta functions have Euler product formulas over the set of periodic orbits.

D. Ruelle, "Zeta functions and statistical mechanics", *Asterisque* **40** (1976), 167-176.

Witten zeta function

A. Reznikov, "Characteristic classes in symplectic topology", *Selecta Math.* vol 3 (1997) 601-642

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