CS888: Introduction to Profession and Communication Skills
-- Theoretical CS

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[*WITH HELP FROM INTERNET SOURCES]

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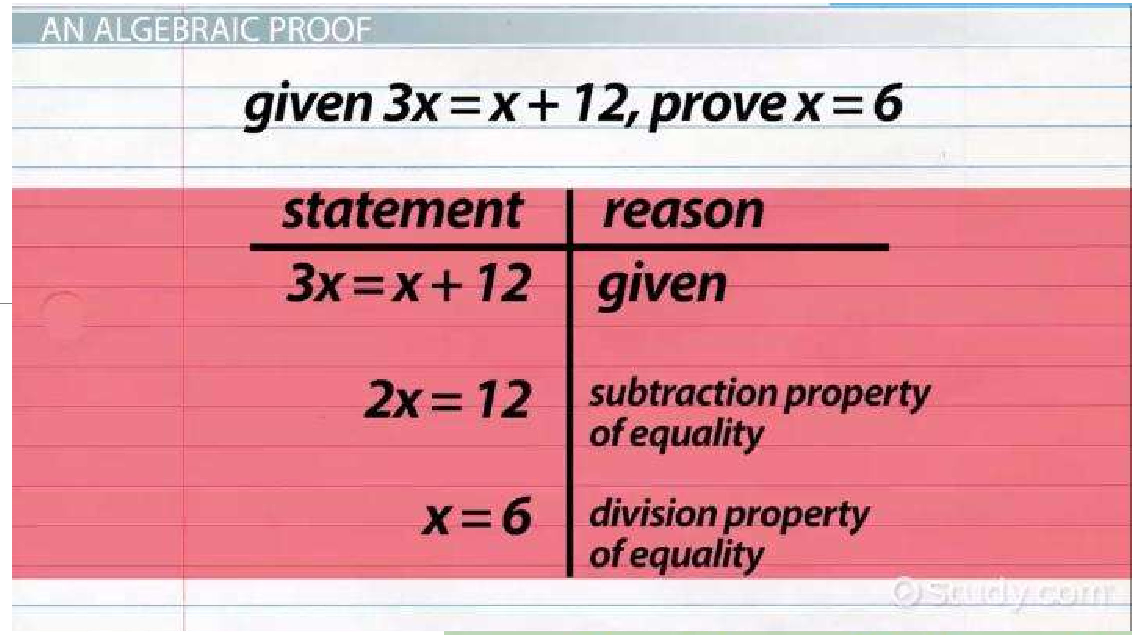
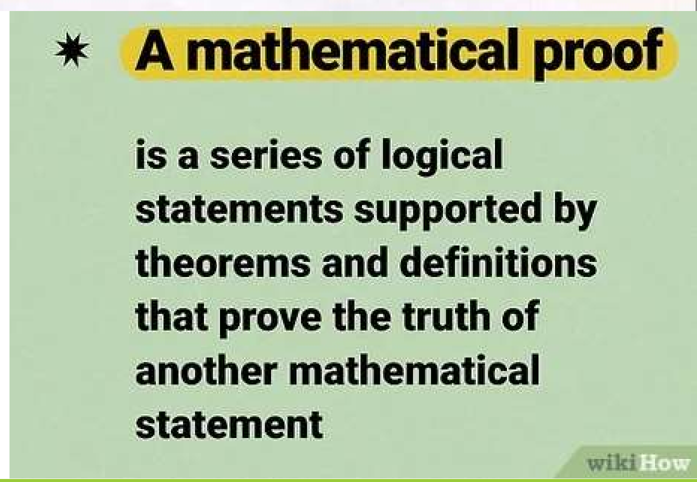
Prove your beliefs

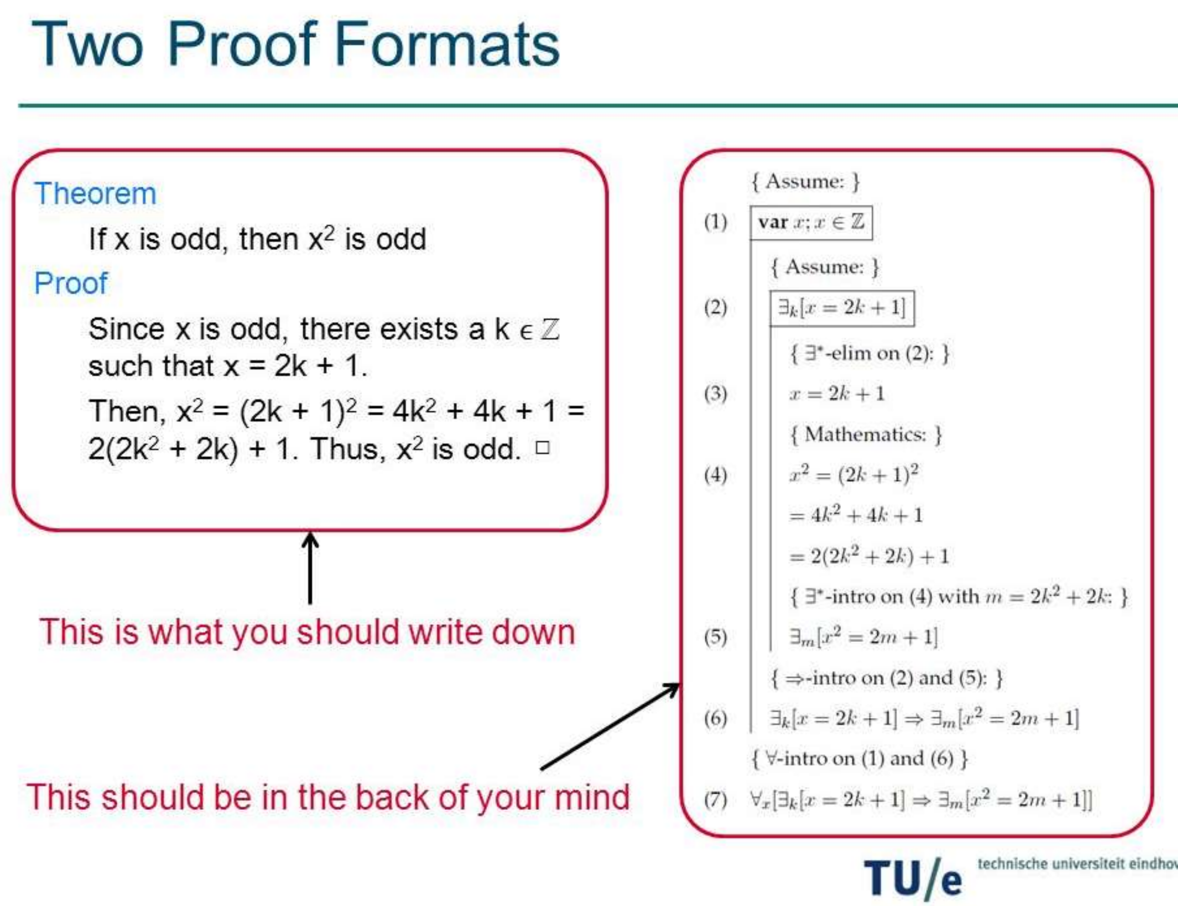
- Maths is written in a proof format.
- Begin: with what you are given.
- *Middle: is direct logical consequence.*
- End: with what was asked to be proved.
  - QED ■
- Break up into *many* Theorems.





Prove your beliefs

- Proof is like a computer program.
- But, keep it readable
  - as much as possible!
- Figure on the right: Steps 1-7 show the logical evolution of statements.
  - much like simple steps of a computer
- Figure on the left: The overall idea gets conveyed with minimal notation.



Fake proofs

- Cancellation rule?
- Caution: Never divide by 0.
  - $\frac{1}{x}$ may be undefined in your ring.

$$
\begin{aligned}
a &= b \\
a^2 &= ab \\
a^2 - b^2 &= ab - b^2 \\
(a+b)(a-b) &= b(a-b) \\
a+b &= b \\
2b &= b \\
2 &= 1
\end{aligned}
$$

Fake proofs

- Product rule of derivative operator.
- The number of summands *non*-constant?
- Caution: Look at the whole function $x * x$
  - sum x "x times" is undefined, for real x

$$x^2 = \underbrace{x + x + x + \cdots + x}_{x \text{ times}}$$

$$\frac{d}{dx}(x^2) = \frac{d}{dx}(\underbrace{x + x + x + \cdots + x}_{x \text{ times}})$$

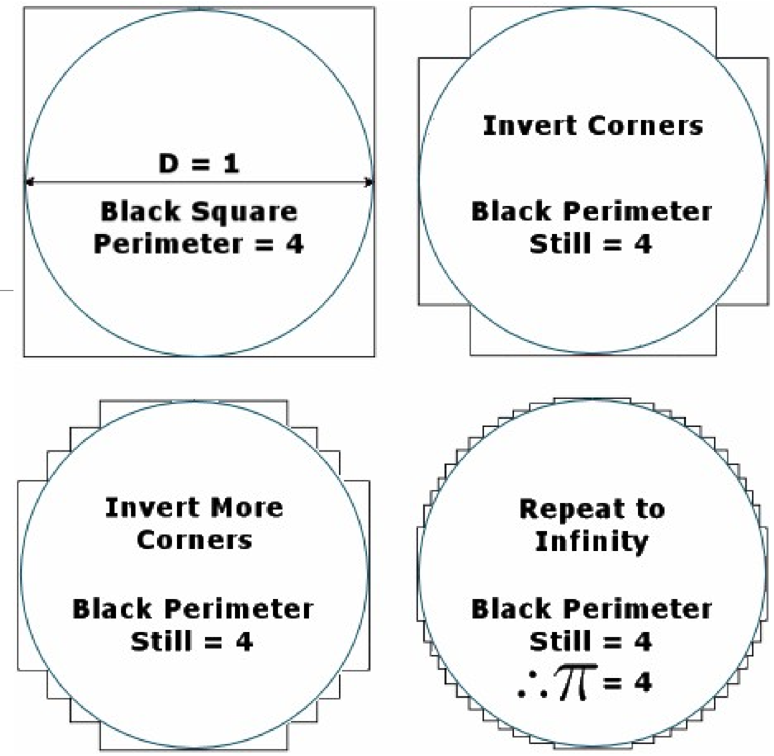
$$2x = \underbrace{1 + 1 + 1 + \cdots + 1}_{x \text{ times}}$$

$$2x = x$$

$$2 = 1$$

Fake proofs

- Perimeter = sum of parts.
- Infinitely many parts?
- Do the parts *converge* to the circle?
- Where's the *error* converging to?
- Caution: Part should be a *tangent*.
  - Discretization introduces large errors
  - Bound them!



Fake proofs by AI chatbots

What's a good proof that every prime is the sum of two squares?

One way to prove that every prime number can be written as the sum of two squares is to use the fact that every positive integer can be written as a sum of four squares. This is known as Lagrange's Four Square Theorem.

y positive integer
the sum of four
roceed by induction.
easy to check: 1 can
+ 0^2 + 0^2 + 0^2.
t every positive
l can be written as
uares. We need to
lso be written as the
es.

Can a rectangle and a circle have the same area?

No, a rectangle and a circle cannot have the same area unless the rectangle is not a true rectangle, but a shape that has curved sides and resembles a circle.

This is because the area of a rectangle is given by the formula A = l x w, where l represents the length of the rectangle, and w represents its width. On the other hand, the area of a circle is given by the formula A = πr^2, where π is the mathematical constant pi and r represents the radius of the circle.

Since the area of a rectangle is always a product of two positive numbers, whereas the area of a circle is always a product of π and the square of the circle's radius, it is impossible for them to have the same area.

Even if we take the square of the circle's radius to be a rational number, π will still be an irrational number, making it impossible for the areas to match exactly.

To prove that $(27)^{1/3}$ is irrational, we can use proof by contradiction.

Let's assume that $(27)^{1/3}$ is rational. This means it can be expressed as a ratio of two integers in the form $\frac{a}{b}$, where a and b are integers with no common factors other than 1, and b is not equal to 0.

So, we have $(27)^{1/3} = \frac{a}{b}$, where a and b have no common factors other than 1.

Now, let's cube both sides of the equation:

$$(27)^{1/3} = \left(\frac{a}{b}\right)^3$$
$$27 = \frac{a^3}{b^3}$$

Multiplying both sides by b^3, we get:

Assignment 7

https://hello.iitk.ac.in/

deadline 12pm (end of class)