

CS888: Introduction to Profession and Communication Skills -- Theoretical CS

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[*WITH HELP FROM INTERNET SOURCES]

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Prove your beliefs

- ❖ Maths is written in a **proof** format.
- ❖ Begin: with what you are given.
- ❖ *Middle: is direct logical consequence.*
- ❖ End: with what was asked to be proved.
 - ❖ QED ■
- ❖ Break up into *many* Theorems.

AN ALGEBRAIC PROOF	
given $3x = x + 12$, prove $x = 6$	
statement	reason
$3x = x + 12$	given
$2x = 12$	subtraction property of equality
$x = 6$	division property of equality

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QED

means
quod erat demonstrandum (which was to be proved or demonstrated)

by acronymsandslang.com

* **A mathematical proof**

is a series of logical statements supported by theorems and definitions that prove the truth of another mathematical statement

wikiHow

Prove your beliefs

- ❖ Proof is like a computer program.
- ❖ But, keep it **readable**
 - ❖ as much as possible!
- ❖ Figure on the right: Steps 1-7 show the **logical evolution** of statements.
 - ❖ much like **simple steps** of a computer
- ❖ Figure on the left: The overall idea gets conveyed with **minimal notation**.

Two Proof Formats

Theorem

If x is odd, then x^2 is odd

Proof

Since x is odd, there exists a $k \in \mathbb{Z}$ such that $x = 2k + 1$.

Then, $x^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$. Thus, x^2 is odd. \square

This is what you should write down

This should be in the back of your mind

```
{ Assume: }
(1) var x; x ∈ ℤ
    { Assume: }
(2)  ∃k[x = 2k + 1]
    { ∃*-elim on (2): }
(3)  x = 2k + 1
    { Mathematics: }
(4)  x2 = (2k + 1)2
      = 4k2 + 4k + 1
      = 2(2k2 + 2k) + 1
    { ∃*-intro on (4) with m = 2k2 + 2k: }
(5)  ∃m[x2 = 2m + 1]
    { ⇒-intro on (2) and (5): }
(6)  ∃k[x = 2k + 1] ⇒ ∃m[x2 = 2m + 1]
    { ∀-intro on (1) and (6): }
(7)  ∀x[∃k[x = 2k + 1] ⇒ ∃m[x2 = 2m + 1]]
```

Fake proofs

- ❖ Cancellation rule?
- ❖ Caution: Never divide by 0.
 - ❖ $\frac{1}{x}$ may be undefined in your ring.

$$\begin{aligned}a &= b \\a^2 &= ab \\a^2 - b^2 &= ab - b^2 \\(a + b)(a - b) &= b(a - b) \\a + b &= b \\2b &= b \\2 &= 1\end{aligned}$$

Fake proofs

- ❖ Product rule of derivative operator.
- ❖ The number of summands *non-constant*?
- ❖ Caution: Look at the whole function $x * x$
 - ❖ sum x “ x times” is undefined, for real x

$$x^2 = \underbrace{x + x + x + \dots + x}_{x \text{ times}}$$

$$\frac{d}{dx}(x^2) = \frac{d}{dx} \underbrace{(x + x + x + \dots + x)}_{x \text{ times}}$$

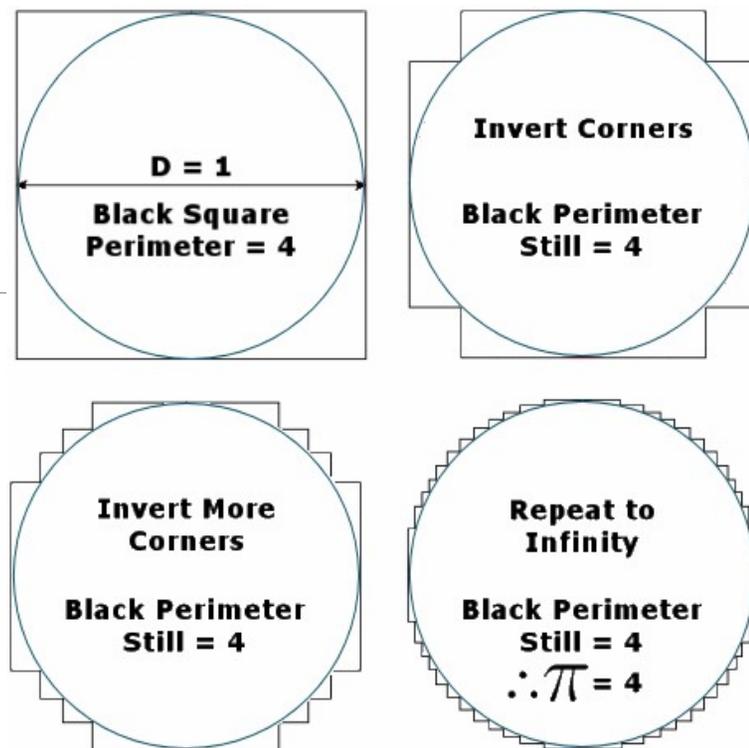
$$2x = \underbrace{1 + 1 + 1 + \dots + 1}_{x \text{ times}}$$

$$2x = x$$

$$2 = 1$$

Fake proofs

- ❖ Perimeter = sum of parts.
- ❖ **Infinitely many parts?**
- ❖ **Do the parts *converge* to the circle?**
- ❖ **Where's the *error* converging to?**
- ❖ **Caution:** Part should be a *tangent*.
 - ❖ Discretization introduces large errors
 - ❖ Bound them!



Fake proofs by AI chatbots



What's a good proof that every prime is the sum of two squares?



One way to prove that every prime number can be written as the sum of two squares is to use the fact that every positive integer can be written as a sum of four squares. This is known as Lagrange's Four Square Theorem.



To prove that $(27)^{1/3}$ is irrational, we can use proof by contradiction.

Let's assume that $(27)^{1/3}$ is rational. This means it can be expressed as a ratio of two integers in the form $\frac{a}{b}$, where a and b are integers with no common factors other than 1, and b is not equal to 0.

So, we have $(27)^{1/3} = \frac{a}{b}$, where a and b have no common factors other than 1.

Now, let's cube both sides of the equation:

$$(27)^{1/3} = \left(\frac{a}{b}\right)^3$$
$$27 = \frac{a^3}{b^3}$$

Multiplying both sides by b^3 , we get:



M

Can a rectangle and a circle have the same area?



No, a rectangle and a circle cannot have the same area unless the rectangle is not a true rectangle, but a shape that has curved sides and resembles a circle.

This is because the area of a rectangle is given by the formula $A = l \times w$, where l represents the length of the rectangle, and w represents its width. On the other hand, the area of a circle is given by the formula $A = \pi r^2$, where π is the mathematical constant pi and r represents the radius of the circle.

Since the area of a rectangle is always a product of two positive numbers, whereas the area of a circle is always a product of π and the square of the circle's radius, it is impossible for them to have the same area.

Even if we take the square of the circle's radius to be a rational number, π will still be an irrational number, making it impossible for the areas to match exactly.

any positive integer
the sum of four
proceed by induction.
easy to check: 1 can
 $+ 0^2 + 0^2 + 0^2$.
every positive
can be written as
squares. We need to
also be written as the
es.



Assignment 7

<https://hello.iitk.ac.in/>
deadline 12pm (end of class)