
"You spelled garbage wrong."
amitabha mukerjee iit kanpur

## Introduction

$\square$ Reading:
Chapter 5 of Jurafsky \& Martin, Speech and Language Processing (2000 edition)
$\square$ Online Coursera lecture:
http://opencourseonline.com/213/stanford-university-nature-language-processing-video-playlist-5-spelling-correction

## Spelling Correction

In [2], the authors used curvatures for accurate loacation and tracking of the center of the eye.

OpenCV has cascades for faces whih have been used for detcting faces in live videos.

- course project report 2013
black crows gorge on bright mangoes in still, dustgreen trees
$\rightarrow$ ?? "black cows" ?? "black crews" ??


## Single-typing errors

$\square$ loacation : insertion error
$\square$ whih, detcting : deletion
$\square$ crows -> crews : substitution
$\square$ the -> hte : transposition

Damereau (1964) : 80\% of all misspelled words caused by single-error of these four types

Which errors have a higher "edit-distance"?

## Causes of Spelling Errors

$\square$ Keyboard Based
$\square 83 \%$ novice and $51 \%$ overall were keyboard related errors

- Immediately adjacent keys in the same row of the keyboard ( $50 \%$ of the novice substitutions, $31 \%$ of all substitutions)
$\square$ Cognitive : may be more than 1-error; more likely to be real words
$\square$ Phonetic: separate $\rightarrow$ separate
$\square$ Homonym : piece $\rightarrow$ peace ; there $\rightarrow$ their;


## Steps in spelling correction

Non-word errors:
$\square$ Detection of non-words (e.g. hte, dtection)
$\square$ Isolated word error correction
[naive bayesian; edit distances]

Actual word (real-word) errors:
$\square$ Context dependent error detection and correction (e.g. "three are four types of errors")
[can use language models e.g. n-grams]

## Probabilistic Spell Checker



Given t , find most probable w :
Find that $\hat{w}$ for which $P(w / t)$ is maximum,


## Probabilistic Spell Checker

$\square \mathrm{Q}$. How to compute $P(w / t)$ ?
$\square$ Many times, it is easier to compute $P(t / w)$

## Bayesian Classification

$\square$ Given an observation $x$, determine which class w it belongs to
$\square$ Spelling Correction:
$\square$ Observation: String of characters
$\square$ Classification: Word intended
$\square$ Speech Recognition:
$\square$ Observation: String of phones
$\square$ Classification: Word that was said

## PROBABILITY THEORY

## Probability theory

Apples and Oranges


## Sample Space

Sample $\omega=$ Pick two fruits,
e.g. Apple, then Orange

Sample Space $\Omega=\{(\mathrm{A}, \mathrm{A}),(\mathrm{A}, \mathrm{O})$, (O,A),(O,O)\}
= all possible worlds

Event $\mathrm{e}=$ set of possible worlds, $\mathrm{e} \subseteq \Omega$

- e.g. second one picked is an apple


## Learning = discovering regularities

- Regularity : repeated experiments: outcome not be fully predictable
- Probability p(e) : "the fraction of possible worlds in which e is true" i.e. outcome is event e
- Frequentist view : $\mathrm{p}(\mathrm{e})=$ limit as $\mathrm{N} \rightarrow \infty$
- Belief view: in wager : equivalent odds
$(1-p): p$ that outcome is in $e$, or vice versa


## Why probability theory?

different methodologies attempted for uncertainty:

- Fuzzy logic
- Multi-valued logic
- Non-monotonic reasoning

But unique property of probability theory:
If you gamble using probabilities you have the best chance in a wager. [de Finetti 1931]
=> if opponent uses some other system, he's more likely to lose

## Ramsay-diFinetti theorem (1931)

If agent X's degrees of belief are rational, then X's degrees of belief function defined by fair betting rates is (formally) a probability function

Fair betting rates: opponent decides which side one bets on

Proof: fair odds result in a function pr () that satisifies the Kolmogrov axioms:

Normality : $\operatorname{pr}(S)>=0$
Certainty : $\operatorname{pr}(\mathrm{T})=1$
Additivity : pr (S1 v S2 v.. $)=\Sigma(\mathrm{Si})$

## Axioms of Probability

- non-negative : $p(e) \geq 0$
- unit sum $p(\Omega)=1$
i.e. no outcomes outside sample space
- additive : if e1, e2 are disjoint events (no common outcome):

$$
p(e 1)+p(e 2)=p(e 1 \cup e 2)
$$

## Joint vs. conditional probability



Marginal Probability

$$
p\left(X=x_{i}\right)=\frac{c_{i}}{N} .
$$

Joint Probability

$$
p\left(X=x_{i}, Y=y_{j}\right)=\frac{n_{i j}}{N}
$$

## Conditional Probability

$$
p\left(Y=y_{j} \mid X=x_{i}\right)=\frac{n_{i j}}{c_{i}}
$$

## Probability Theory



## Sum Rule

$$
\begin{aligned}
& p\left(X=x_{i}\right)=\frac{c_{i}}{N}=\frac{1}{N} \sum_{j=1}^{L} n_{i j} \\
& \quad=\sum_{j=1}^{L} p\left(X=x_{i}, Y=y_{j}\right)
\end{aligned}
$$

Product Rule

$$
\begin{aligned}
p\left(X=x_{i}, Y=y_{j}\right) & =\frac{n_{i j}}{N}=\frac{n_{i j}}{c_{i}} \cdot \frac{c_{i}}{N} \\
& =p\left(Y=y_{j} \mid X=x_{i}\right) p\left(X=x_{i}\right)
\end{aligned}
$$

## Rules of Probability

Sum Rule

$$
p(X)=\sum_{Y} p(X, Y)
$$

Product Rule

$$
p(X, Y)=p(Y \mid X) p(X)
$$

## Example

AIDS (disease d) occurs in $0.05 \%$ of population. A new test is $99 \%$ effective in detecting AIDS, but $5 \%$ of the cases test positive even without AIDS.
10000 people are tested. How many are expected to test positive?

$$
\begin{array}{rlrl}
\mathrm{p}(\mathrm{~d}) & =0.0005 ; p(\mathrm{t} / \mathrm{d})=0.99 ; & \mathrm{p}(\mathrm{t} / \sim \mathrm{d})=0.05 \\
\mathrm{p}(\mathrm{t}) & =\mathrm{p}(\mathrm{t}, \mathrm{~d})+\mathrm{p}(\mathrm{t}, \sim \mathrm{~d}) & & {[\text { Sum Rule }]} \\
& =\mathrm{p}(\mathrm{t} / \mathrm{d}) \mathrm{p}(\mathrm{~d})+\mathrm{p}(\mathrm{t} / \sim \mathrm{d}) \mathrm{p}(\sim \mathrm{~d}) & & {[\text { Product Rule }]} \\
& =0.99 * 0.0005+0.05 * 0.9995=0.0505 \rightarrow 505+\mathrm{ve}
\end{array}
$$

## Probabilistic Spell Checker

$\square \mathrm{Q}$. How to compute $P(w / t)$ ?
$\square$ Many times, it is easier to compute $P(t / w)$
$\square$ Related by product rule:

$$
\begin{aligned}
p(X, Y) & =p(Y \mid X) p(X) \\
& =p(X \mid Y) p(Y)
\end{aligned}
$$

## Bayes' Theorem

$$
\begin{aligned}
p(Y \mid X) & =\frac{p(X \mid Y) p(Y)}{p(X)} \\
p(X) & =\sum_{Y} p(X \mid Y) p(Y)
\end{aligned}
$$

posterior $\propto$ likelihood $\times$ prior

## Bayes' Theorem

Thomas Bayes (c.1750):
how can we infer causes from effects?
how can one learn the probability of a future event from how many times it had (or had not) occurred in the past?
as new evidence comes in $\rightarrow$ probabilistic knowledge improves.
e.g. throw a die. guess is poor (1/6)
throw die again. is it > or < than prev? Can improve guess.
throw die repeatedly. can improve prob of guess quite a lot.
Hence: initial estimate (prior belief $P(h)$, not well formulated)

+ new evidence (support) - compute likelihood $P$ (datal $h$ )
$\rightarrow$ improved estimate (posterior): $P$ (h/ data)


## Example

A disease $d$ occurs in $0.05 \%$ of population. A test is $99 \%$ effective in detecting the disease, but $5 \%$ of the cases test positive in absence of $d$.
If you are tested +ve, what is the probability you have the disease?

$$
\begin{aligned}
& p(d / t)=p(d) \cdot p(t / d) / p(t) ; p(t)=0.0505 \\
& p(d / t)=0.0005^{*} 0.99 / 0.0505=0.0098 \text { (about } 1 \% \text { ) }
\end{aligned}
$$

if 10 K people take the test, $\mathrm{E}(\mathrm{d})=5$

$$
\mathrm{FPs}=0.05 * 9995=500
$$

$$
\text { TPs }=0.99 * 5=\quad 5 . \quad \rightarrow \quad \text { only } 5 / 505 \text { have } d
$$

## Precision vs Recall

Precision:
A / Retrieved Positives

Recall:
A / Actual
Positives


## Example

What is the recall of the test $t$ ?
What is its precision?
Recall $=$ fraction of actual positives that are detected by $t$ $=0.99$

Precision = \%age of true positives among cases that $t$ finds positive

$$
=5 / 505=.0098
$$

## Features may be high-dimensional




joint distribution $\mathrm{P}(\mathrm{x}, \mathrm{y})$ varies considerably though marginals $\mathrm{P}(\mathrm{x}), \mathrm{P}(\mathrm{y})$ are identical
estimating the joint distribution requires much larger sample: $O\left(n^{k}\right)$ vs $n k$

NON-WORD SPELL CHECKER

## Spelling error as classification

$\square$ Each word $w$ is a class, related to many instances of the observed forms $x$
$\square$ Assign w given x :

$$
\hat{w}=\underset{w V}{\operatorname{argmax}} P(w \mid x)
$$

## Noisy Channel : Bayesian Modeling

$\square$ Observation x of a misspelled word
$\square$ Find correct word w

$$
\begin{aligned}
\hat{w} & =\underset{w V}{\operatorname{argmax}} P(w \mid x) \\
& =\underset{w V}{\operatorname{argmax}} \frac{P(x \mid w) P(w)}{P(x)} \\
& =\underset{V}{\operatorname{argmax}} P(x \mid w) P(w)
\end{aligned}
$$

## Non-word spelling error example

acress

## Confusion Set

Confusion set of word w:
All typed forms t obtainable by a single application of insertion, deletion, substitution or transposition

## Confusion set for acress

| Error | Candidate <br> Correction | Correct <br> Letter | Error <br> Letter |  |
| :--- | :--- | :--- | :--- | :--- |
| acress | actress | t | - | deletion |
| acress | cress | - | a | insertion |
| acress | caress | ca | ac | transposition |
| acress | access | c | r | substitution |
| acress | across | o | e | substitution |
| acress | acres | - | s | insertion |
| acress | acres | - | s | insertion |

## Kernighan et al 90

Confusion set of word w (one edit operation away from w):
$\square$ All typed forms tobtainable by a single application of insertion, deletion, substitution or transposition

Different editing operations have unequal weights Insertion and deletion probabilities : conditioned on letter immediately on the left - bigram model.

Compute probabilities based on training corpus of single-typing errors.

## Unigram Prior probability

Counts from 404,253,213 words in Corpus of Contemporary English (COCA)

| word | Frequency of <br> word | $\mathrm{P}($ word $)$ |
| :--- | ---: | :--- |
| actress | 9,321 | .0000230573 |
| cress | 220 | .0000005442 |
| caress | 686 | .0000016969 |
| access | 37,038 | .0000916207 |
| across | 120,844 | .0002989314 |
| acres | 12,874 | .0000318463 |

## Channel model probability

$\square$ Error model probability, Edit probability
$\square$ Kernighan, Church, Gale 1990
$\square$ Misspelled word $x=x_{1}, x_{2}, x_{3} \ldots x_{m}$
$\square$ Correct word $w=w_{1}, w_{2}, w_{3}, \ldots, w_{n}$
$\square \mathrm{P}(\mathrm{x} \mid \mathrm{w})=$ probability of the edit
$\square$ (deletion/insertion/substitution/transposition)

## Computing error probability: confusion matrix

del[ $x, y$ ]: $\quad$ count ( $x y$ typed as $x$ ) ins[ $x, y$ ]: count( $x$ typed as $x y$ )
sub[ $x, y$ ]: count( $x$ typed as $y$ ) trans[x,y]: count(xy typed as $y x$ )

Insertion and deletion conditioned on previous character

## Confusion matrix - Deletion [Kerni90]

 Y (Deteted Letter)| X | $Y$ (Deteted Letter) |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | a | b | c | d | e | f | g | h i | j | k | 1 | m | n | 0 | $p$ | 9 | $r$ | $s$ | $t$ | u | $v$ | w | x | y | z |
| a | 0 | 7 | 58 | 21 | 3 | 5 | 18 | 861 | 0 | 4 | 43 | 5 | 53 | 0 | 9 | 0 | 98 | 28 | 53 | 62 | 1 | 0 | 0 | 2 | 0 |
| b | 2 | 2 | 1 | 0 | 22 | 0 | 0 | 0183 | 0 | 0 | 26 | 0 | 0 | 2 | 0 | 0 | 6 | 17 | 0 | 6 | 1 | 0 | 0 | 0 |  |
| c | 37 | 0 | 70 | 0 | 63 | 0 | 0 | 24320 | 0 | 9 | 17 | 0 | 0 | 33 | 0 | 0 | 46 | 6 | 54 | 17 | 0 | 0 | 0 | 1 |  |
| d | 12 | 0 | 7 | 25 | 45 | 0 | 10 | 062 | 1 | 1 | 8 | 4 | 3 | 3 | 0 | 0 | 11 | 1 | 0 | 3 | 2 | 0 | 0 | 6 |  |
| e | 80 | 1 | 50 | 74 | 89 | 3 | 1 | 16 | 0 | 0 | 32 | 9 | 76 | 19 | 9 | 1 | 237 | 223 | 34 | 8 | 2 | 1 | 7 | 1 |  |
| f | 4 | 0 | 0 | 0 | 13 | 46 | 0 | 079 | 0 | 0 | 12 | 0 | 0 | 4 | 0 | 0 | 11 | 0 | 8 | 1 | 0 | 0 | 0 | 1 |  |
| $g$ | 25 | 0 | 0 | 2 | 83 | 1 | 37 | $25 \quad 39$ | 0 | 0 | 3 | 0 | 29 | 4 | 0 | 0 | 52 | 7 | 1 | 22 | 0 | 0 | 0 | 1 |  |
| h | 15 | 12 | 1 | 3 | 20 | 0 | 0 | $25 \quad 24$ | 0 | 0 | 7 | 1 | 9 | 22 | 0 | 0 | 15 | 1 | 26 | 0 | 0 | 1 | 0 | 1 |  |
| i | 26 | 1 | 60 | 26 | 23 | 1 | 9 | 01 | 0 | 0 | 38 | 14 | 82 | 41 | 7 | 0 | 16 | 71 | 64 | 1 | 1 | 0 | 0 | 1 |  |
| j | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 00 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |  |
| k | 4 | 0 | 0 | 1 | 15 | 1 | 8 | 15 | 0 | 1 | 3 | 0 | 17 | 0 | 0 | 0 | 1 | 5 | 0 | 0 | 0 | 1 | 0 | 0 |  |
| 1 | 24 | 0 | 1 | 6 | 48 | 0 | 0 | 0217 | 0 | 0 | 211 | 2 | 0 | 29 | 0 | 0 | 2 | 12 | 7 | 3 | 2 | 0 | 0 | 11 |  |
| m | 15 | 10 | 0 | 0 | 33 | 0 | 0 | 142 | 0 | 0 | 0 | 180 | 7 | 7 | 31 | 0 | 0 | 9 | 0 | 4 | 0 | 0 | 0 | 0 |  |
| n | 21 | 0 | 42 | 71 | 68 | 1 | 160 | 0191 | 0 | 0 | 0 | 17 | 144 | 21 | 0 | 0 | 0 | 127 | 87 | 43 | 1 | 1 | 0 | 2 |  |
| o | 11 | 4 | 3 | 6 | 8 | 0 | 5 | 04 | 1 | 0 | 13 | 9 | 70 | 26 | 20 | 0 | 98 | 20 | 13 | 47 | 2 | 5 | 0 | 1 |  |
| p | 25 | 0 | 0 | 0 | 22 | 0 | 0 | 1215 | 0 | 0 | 28 | 1 | 0 | 30 | 93 | 0 | 58 | 1 | 18 | 2 | 0 | 0 | 0 | 0 |  |
| q | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 18 | 0 | 0 | 0 | 0 |  |
| r | 63 | 4 | 12 | 19 | 188 | 0 | 11 | 5132 | 0 | 3 | 33 | 7 | 157 | 21 | 2 | 0 | 277 | 103 | 68 | 0 | 10 | 1 | 0 | 27 |  |
| s | 16 | 0 | 27 | 0 | 74 | 1 | 0 | 18231 | 0 | 0 | 2 | 1 | 0 | 30 | 30 | 0 | 4 | 265 | 124 | 21 | 0 | 0 | 0 | 1 |  |
| $t$ | 24 | 1 | 2 | 0 | 76 | 1 | 7 | 49427 | 0 | 0 | 31 | 3 | 3 | 11 | 1 |  | 203 | 5 | 137 | 14 | 0 | 4 | 0 | 2 |  |
| u | 26 | 6 | 9 | 10 | 15 | 0 | 1 | 028 | 0 | 0 | 39 | 2 | 111 | 1 | 0 | 0 | 129 | 31 | 66 | 0 | 0 | 0 | 0 | 1 |  |
| v | 9 | 0 | 0 | 0 | 58 | 0 | 0 | 031 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |  |
| w | 40 | 0 | 0 | 1 | 11 | 1 | 0 | 1115 | 0 | 0 | 1 | 0 | 2 | 2 | 0 | 0 | 2 | 24 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| x | 1 | 0 | 17 | 0 | 3 | 0 | 0 | 10 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 0 | 0 | 0 | 5 | 0 | 0 | 0 | 0 | 1 |  |
| y | 2 | 1 | 34 | 0 | 2 | 0 | 1 | 01 | 0 | 0 | 1 | 2 | 1 | 1 | 1 | 0 | 0 | 17 | 1 | 0 | 0 | 1 | 0 | 0 |  |
| $z$ | 1 | 0 | 0 | 0 | 2 | 0 | 0 | 00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| @ | 20 | 14 | 41 | 31 | 20 | 20 | 7 | 620 | 3 | 6 | 22 | 16 | 5 | 5 | 17 | 0 | 28 | 26 | 6 | 2 | 1 | 24 | 0 | 0 |  |

## Confusion matrix : substitution

$\operatorname{sub}[\mathbf{X}, \mathrm{Y}]=$ Substitution of $\mathbf{X}$ (incorrect) for $\mathbf{Y}$ (correct)

| X |  |  |  |  |  |  |  |  |  |  |  | (c) | t) |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | a | b | c | d | $f$ | g | h | i | j | k | 1 | m | n | 0 | p | 9 | r | s | $t$ | u | $v$ | w | x | y |  |
| a | 0 | 0 | 7 | 1342 | 0 | 0 | 21 | 118 | 0 | 1 | 0 | 0 | 3 | 76 | 0 | 0 | 1 | 35 | 9 | 9 | 0 | 1 | 0 | 5 | ) |
| b | 0 | 0 | 9 | 92 | 2 | 3 | 1 | 0 | 0 | 0 | 5 | 11 | 5 | 0 | 10 | 0 | 0 | 2 | 1 | 0 | 0 | 8 | 0 | 0 |  |
| c | 6 | 5 | 0 | 160 | 9 | 5 | 0 | 0 | 0 | 1 | 0 | 7 | 9 | 1 | 10 | 2 | 5 | 39 | 40 | 1 | 3 | 7 | 1 | 1 |  |
| d | 1 | 10 | 13 | 012 | 0 | 5 | 5 | 0 | 0 | 2 | 3 | 7 | 3 | 0 | 1 | 0 | 43 | 30 | 22 | 0 | 0 | 4 | 0 | 2 |  |
| c | 388 | 0 | 3 | 110 | 2 | 2 | 0 | 89 | 0 | 0 | 3 | 0 | 5 | 93 | 0 | 0 | 14 | 12 | 6 | 15 | 0 | 1 | 0 | 18 | 0 |
| $f$ | 0 | 15 | 0 | 3 | 0 | 5 | 2 | 0 | 0 | 0 |  | 4 | 1 | 0 | 0 | 0 | 6 | 4 | 12 | 0 | 0 | 2 | 0 | 0 | ) |
| g | 4 | 1 | 11 | 11 | 2 | 0 | 0 | 0 | 1 | 1 | 3 | 0 | 0 | 2 | 1 | 3 | 5 | 13 | 21 | 0 | 0 | 1 | 0 | 3 |  |
| h | 1 | 8 | 0 | 30 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 12 | 14 | 2 | 3 | 0 | 3 | 1 | 11 | 0 | 0 | 2 | 0 | 0 | 0 |
| i | 103 | 0 | 0 | 0146 | 0 | 1 | 0 | 0 | 0 | 0 | 6 | 0 | 0 | 49 | 0 | 0 | 0 | 2 | 1 | 47 | 0 | 2 | 1 | 15 | 0 |
| j | 0 | 1 | 1 | 90 | 0 | 1 | 0 | 0 | 0 | 0 | 2 | 1 | 0 | 0 | 0 | 0 | 0 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| k | 1 | 2 | 8 | 41 | 1 | 2 | 5 | 0 | 0 | 0 | 0 | 5 | 0 | 2 | 0 | 0 | 0 | 6 | 0 | 0 | 0 | 4 | 0 | 0 | 3 |
| 1 | 2 | 10 | 1 | 40 | 4 | 5 | 6 | 13 | 0 | 1 | 0 | 0 | 14 | 2 | 5 | 0 | 11 | 10 | 2 | 0 | 0 | 0 | 0 | 0 |  |
| m | 1 | 3 | 7 | 80 | 2 | 0 | 6 | 0 | 0 | 4 | 4 |  | 180 | 0 | 6 | 0 | 0 | 9 | 15 | 13 |  | 2 | 2 | 3 |  |
| n | 2 | 7 | 6 | 53 | 0 | 1 | 19 | 1 | 0 | 4 | 35 | 78 | 0 | 0 | 7 | 0 | 28 | , | 7 | ) | 0 | 1 | 2 | 0 | 2 |
| 0 | 91 | 1 | 1 | 3116 | 0 | 0 | 0 | 25 | 0 | 2 | 0 | 0 | 0 | 0 | 14 | 0 | 2 | 4 | 14 | 39 | 0 | 0 | 0 | 18 |  |
| p | 0 | 11 | 1 | 20 | 6 | 5 | 0 | 2 | 9 | 0 | 2 | 7 | 6 | 15 | 0 | 0 |  | 3 | 6 | 0 | 4 | 1 | 0 | 0 |  |
| q | 0 | 0 | 1 | 0 | 0 | 27 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | ) |
| r | 0 | 14 | 0 | $30 \quad 12$ | 2 | 2 |  | 2 | 0 | 5 |  | 4 | 20 | 1 | 14 | 0 | 0 | 12 | 22 |  | 0 | 0 | 1 | 0 |  |
| s | 11 | 8 | 27 | 3335 | 4 | 0 | 1 | 0 | 1 | 0 | 27 | 0 | 6 |  |  | 0 | 14 | 0 | 15 | 0 | 0 | 5 | 3 | 20 |  |
| $t$ | 3 | 4 | 9 | 427 | 5 | 19 | 5 | 0 | 1 | 0 | 14 | 9 | 5 | 5 | 6 | 0 | 11 | 37 | 0 | 0 | 2 | 19 | 0 | 7 |  |
| u | 20 | 0 | 0 | 044 | 0 | 0 | 0 | 64 | 0 | 0 | 0 | 0 | 2 | 43 | 0 | 0 | 4 | 0 |  | 0 | 0 | 2 | 0 | 8 |  |
| $v$ | 0 | 0 | 7 | $0 \quad 0$ | 3 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |  | 8 |  | 0 | 0 | 0 | 0 | 0 |  |
| w | 2 | 2 | 1 | 01 | 0 | 0 | 2 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 7 | 0 | 6 | 3 | 3 | 1 | 0 | 0 | 0 | 0 |  |
| x | 0 | 0 | 0 | 20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 9 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| y | 0 | 0 | 2 | 015 | 0 | 1 | 7 | 15 | 0 | 0 | 0 | 2 | 0 | 6 | 1 | 0 | 7 | 36 | 8 | 5 | 0 | 0 | 1 | 0 | $0$ |
|  | 0 | 0 | 0 | 70 | 0 | 0 | 0 | 0 | 0 | 0 | 7 | 5 | 0 | 0 | 0 | 0 | 2 | 21 | 3 | 0 | 0 | 0 | 0 | 3 |  |

## Channel model

## Channel model for acress

| Candidate | Correct | Error <br> Correction | $\mathrm{x} \mid \mathrm{w}$ | $\mathrm{P}(\mathrm{x} \mid$ word $)$ |
| :--- | :--- | :--- | :--- | :--- |
| Letter | Letter |  |  |  |
| actress | t | - | $\mathrm{c} \mid \mathrm{ct}$ | .000117 |
| cress | - | a | $\mathrm{a} \mid \#$ | .00000144 |
| caress | ca | ac | $\mathrm{ac} \mid \mathrm{ca}$ | .00000164 |
| access | c | r | $\mathrm{r} \mid \mathrm{c}$ | .000000209 |
| across | o | e | $\mathrm{e} \mid \mathrm{o}$ | .0000093 |
| acres | - | s | $\mathrm{es} \mid \mathrm{e}$ | .0000321 |
| acres | - | s | $\mathrm{ss} \mid \mathrm{s}$ | .0000342 |

## Noisy channel probability for acress

| Candida te Correcti on | Corre ct <br> Letter | Erro r <br> Lett er | $x \mid w$ | $\mathrm{P}(\mathrm{x} \mid$ word) | P(word) | $10^{9}$ * $\mathrm{P}(\mathrm{x} \mid \mathrm{w}) \mathrm{P}(\mathrm{w})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| actress | t | - | $\mathrm{c} \mid \mathrm{ct}$ | . 000117 | . 0000231 | 2.7 |
| cress | - | a | a\|\# | . 00000144 | $\begin{aligned} & .00000054 \\ & 4 \end{aligned}$ | . 00078 |

caress ca ac ac|ca . 00000164 . 00000170 . 0028

| access | c | r | $\mathrm{r} \mid \mathrm{c}$ | .000000209 | .0000916 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| .019 |  |  |  |  |  |
| across | o | e | $\mathrm{e} \mid \mathrm{O}$ | .0000093 | .000299 |
| acres | - | s | es\|e | .0000321 | .0000318 |
| acres | - | s | ss\|s | .0000342 | .0000318 |

## Using a bigram language model

"a stellar and versatile acress whose combination of sass and glamour..."$\square$ Counts from the Corpus of Contemporary American English with add-1 smoothing
$\square \mathrm{P}($ actress|versatile $)=.000021$
$\square \mathrm{P}($ across|versatile $)=.000021$
$\mathrm{P}($ whose|actress $)=.0010$
$\mathrm{P}($ whose $\mid$ across $)=.000006$
$\square \mathrm{P}\left(\right.$ "versatile actress whose") $=.000021^{*} .0010=210 \times 10^{-10}$
$\square \mathrm{P}\left(\right.$ "versatile across whose") $=.000021^{*} .000006=1 \times 10^{-10}$

## Multiple Typing Errors

## Multiple typing errors

$\square$ Measures of string similarity
How similar is "intension" to "execution"?
$\square$ For strings of same length - Hamming distance
$\square$ Edit distance (A,B):
minimum number of operations that transform string A into string B
$\square$ ins, del, sub, transp : Damerau -Levenshtein distance

## Minimum Edit Distance

$\square$ Each edit operation has a cost
$\square$ Edit distance based measures
$\square$ Levnishtein-Damreau distance
$\square$ How similar is "intension" to "execution"?

## Three views of edit operations

Alignment
All views $\rightarrow$
cost $=5$ edits

If subst / transp is not allowed
[their cost $=2$ ] $\rightarrow$
cost= 8 edits
Operation
List

Trace



$$
\begin{aligned}
& i \mathrm{n} t \in \mathrm{n} \varepsilon \mathrm{t} i \circ \mathrm{n} \\
& \varepsilon \in \mathrm{x} e \mathrm{c} u \mathrm{t} \text { i } \circ \mathrm{n}
\end{aligned}
$$

## Levenshtein Distance

$\square \operatorname{len}(A)=m ; \operatorname{len}(B)=n$
$\square$ create $\mathrm{n} \times \mathrm{m}$ matrix : A along x -axis, B along y
$\square \operatorname{cost}(\mathrm{i}, \mathrm{j})=$ Levenshtein distance ( $\mathrm{A}[0 . \mathrm{i}], \mathrm{B}[0 . . \mathrm{j}]$ )
= cost of matching substrings
$\square$ Dynamic programming : solve by decomposition.
$\square$ Dist-matrix $(i, j)=\min \{$ costs of insert from (i-1,j) or (i,j-1 ); or cost of substitute from (i-1, j-1) \}

## Levenshtein Distance

| n | 9 | 10 | 11 | 10 | 11 | 12 | 11 | 10 | 9 | 8 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| o | 8 | 9 | 10 | 9 | 10 | 11 | 10 | 9 | 8 | 9 |
| i | 7 | 8 | 9 | 8 | 9 | 10 | 9 | 8 | 9 | 10 |
| t | 6 | 7 | 8 | 7 | 8 | 9 | 8 | 9 | 10 | 11 |
| n | 5 | 6 | 7 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| e | 4 | 5 | 6 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| t | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| n | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 8 | 10 | 11 |
| i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| \# | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|  | $\#$ | e | x | e | c | u | t | i | o | n |

## WORD-FROM-DICTIONARY SPELL CHECKER

## WORD-FROM-DICTIONARY SPELL CHECKER

## Real-word spelling errors

$\square$...leaving in about fifteen minuets to go to her house.
$\square$ The design an construction of the system...
$\square$ Can they lave him my messages?
$\square$ The study was conducted mainly be John Black.
$\square \mathbf{2 5 - 4 0 \%}$ of spelling errors are real words Kukich 1992

## Solving real-world spelling errors

$\square$ For each word in sentence
$\square$ Generate candidate set

- the word itself
- all single-letter edits that are English words
- words that are homophones
$\square$ Choose best candidates
- Noisy channel model
- Task-specific classifier


## Noisy channel for real-word spell correction

$\square$ Given a sentence $\mathrm{w}_{1}, \mathrm{w}_{2}, \mathrm{w}_{3}, \ldots, \mathrm{w}_{\mathrm{n}}$
$\square$ Generate a set of candidates for each word $w_{i}$
$\square$ Candidate $\left(w_{1}\right)=\left\{w_{1}, w_{1}^{\prime}, w_{1}{ }_{1}, w^{\prime \prime \prime}{ }_{1}, \ldots\right\}$
$\square$ Candidate $\left(w_{2}\right)=\left\{w_{2}, w_{2}^{\prime}, w^{\prime \prime}{ }_{2}, w^{\prime \prime \prime}{ }_{2}, \ldots\right\}$
$\square$ Candidate $\left(w_{n}\right)=\left\{w_{n}, w_{n}^{\prime}, w_{n}^{\prime \prime}, w^{\prime \prime \prime}{ }_{n}, \ldots\right\}$
$\square$ Choose the sequence W that maximizes $\mathrm{P}(\mathrm{W})$

## Noisy channel for real-word spell correction



## Noisy channel for real-word spell correction



## Norvig's Python Spelling Corrector

How to Write a Spelling Corrector
http://norvig.com/spell-correct.html

## Simplification: One error per sentence

$\square$ Out of all possible sentences with one word replaced
$\square w_{1}, w^{\prime \prime}{ }_{2}, w_{3}, w_{4} \quad$ two off thew
$\square w_{1}, w_{2}, w_{3}^{\prime}, w_{4} \quad$ two of the
$\square \mathbf{w}^{\prime \prime \prime}{ }_{1}, W_{2}, W_{3}, W_{4} \quad$ too of thew
$\square$ Choose the sequence W that maximizes $\mathrm{P}(\mathrm{W})$

## Where to get the probabilities

$\square$ Language model

- Unigram
$\square$ Bigram
- Etc
$\square$ Channel model
$\square$ Same as for non-word spelling correction
$\square$ Plus need probability for no error, $\mathrm{P}(\mathrm{w} \mid \mathrm{w})$


## Probability of no error

$\square$ What is the channel probability for a correctly typed word?
$\square \mathrm{P}($ "the" $\mid$ "the" $)=1$ - probability of mistyping
$\square$ Depends on typist, task, etc.
$\square .90$ (1 error in 10 words)
$\square .95$ (1 error in 20 words) $\leftarrow$ value used, say
-. 99 (1 error in 100 words)

- . 995 (1 error in 200 words)


## Peter Norvig's "thew" example

|  |  |  |  | $10^{9}$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{x}$ | w | $\mathrm{x} \mid \mathrm{w}$ | $\mathrm{P}(\mathrm{x} \mid \mathrm{w})$ | $\mathrm{P}(\mathrm{w})$ | $\mathrm{P}(\mathrm{x} \mid \mathrm{w}) \mathrm{P}(\mathrm{w})$ |
| thew | the | ew\|e | 0.000007 | 0.02 | 144 |
| thew | thew |  | 0.95 | 0.00000009 | 90 |

Choosing 0.99 instead of 0.95 ( 1 mistyping in 100 words) $\rightarrow$ "thew" becomes more likely

## State of the art noisy channel

$\square$ We never just multiply the prior and the error model
$\square$ Independence assumptions $\rightarrow$ probabilities not commensurate
$\square$ Instead: weight them

$$
\hat{w}=\underset{w V}{\operatorname{argmax}} P(x \mid w) P(w)
$$

$\square$ Learn $\lambda$ from a validation test set (divide training set into training + validation)

## Phonetic error model

$\square$ Metaphone, used in GNU aspell
$\square$ Convert misspelling to metaphone pronunciation

- "Drop duplicate adjacent letters, except for C."

■ "If the word begins with 'KN', 'GN', 'PN', 'AE', 'WR', drop the first letter."

- "Drop ' B ' if after ' M ' and if it is at the end of the word"
- ...
$\square$ Find words whose pronunciation is 1-2 edit distance from misspelling's
$\square$ Score result list
$\square$ Weighted edit distance of candidate to misspelling
- Edit distance of candidate pronunciation to misspelling pronunciation


## Improvements to channel model

$\square$ Allow richer edits (Brill and Moore 2000)

- ent $\rightarrow$ ant
$\square \mathrm{ph} \rightarrow \mathrm{f}$
$\square \mathrm{le} \rightarrow$ al
$\square$ Incorporate pronunciation into channel (Toutanova and Moore 2002)


## Channel model

$\square$ Factors that could influence p (misspelling |word)
$\square$ The source letter
$\square$ The target letter
$\square$ Surrounding letters
$\square$ The position in the word
$\square$ Nearby keys on the keyboard
$\square$ Homology on the keyboard
$\square$ Pronunciations
$\square$ Likely morpheme transformations

Nearby keys


## Classifier-based methods

$\square$ Instead of just channel model and language model
$\square$ Use many more features - wider context build a classifier (machine learning).
$\square$ Example:
whether/weather
■ "cloudy" within +- 10 words

- _ to VERB
- _ or not
$\square$ Q. How can we discover such features?


## Candidate generation

$\square$ Words with similar spelling
$\square$ Small edit distance to error
$\square$ Words with similar pronunciation
$\square$ Small edit distance of pronunciation to error

## Damerau-Levenshtein edit distance

$\square$ Minimal edit distance between two strings, where edits are:
$\square$ Insertion

- Deletion
$\square$ Substitution
$\square$ Transposition of two adjacent letters


## Candidate generation

$\square 80 \%$ of errors are within edit distance 1
$\square$ Almost all errors within edit distance 2
$\square$ Also allow insertion of space or hyphen
$\square$ thisidea $\rightarrow$ this idea
$\square$ inlaw $\rightarrow$ in-law

## Language Model

$\square$ Language modeling algorithms :

- Unigram, bigram, trigram
$\square$ Formal grammars
$\square$ Probabilistic grammars


## FINITE STATE MORPHOLOGY

## Computational morphology

## Analysis



## Generation


hung

## Two challenges

$\square$ Morphotactics
$\square$ Words are composed of smaller elements that must be combined in a certain order:
■ piti-less-ness is English

- piti-ness-less is not English
$\square$ Phonological alternations
$\square$ The shape of an element may vary depending on the context
- pity is realized as piti in pitilessness
- die becomes dy in dying


## Morphology is regular (=rational)

$\square$ The relation between the surface forms of a language and the corresponding lexical forms can be described as a regular relation.
$\square$ A regular relation consists of ordered pairs of strings.

- leaf+N+Pl: leaves hang+V+Past : hung
$\square$ Any finite collection of such pairs is a regular relation.
$\square$ Regular relations are closed under operations such as concatenation, iteration, union, and composition.
- Complex regular relations can be derived from simple relations.


## Morphology is finite-state

$\square$ A regular relation can be defined using the metalanguage of regular expressions.

- [\{talk\} | \{walk\} | \{work\}]
- [\%+Base:0 | \%+SgGen3:s | \%+Progr:\{ing\} |
\%+Past:\{ed\}];
$\square$ A regular expression can be compiled into a finitestate transducer that implements the relation computationally.


## Compilation

## Regular expression

- [\{talk\} | \{walk\} | \{work\}]
- [\%+Base:0 | \%+SgGen3:s | \%+Progr:\{ing\} | \%+Past:\{ed\}];


## Finite-state transducer



## Generation

> work+3rdSg --> works


"You spelled garbage wrona."
amitabha mukerjee iit kanpur

## HCl issues in spelling

$\square$ If very confident in correction
$\square$ Autocorrect
$\square$ Less confident
$\square$ Give the best correction
$\square$ Less confident
$\square$ Give a correction list
$\square$ Unconfident
$\square$ Just flag as an error

## Noisy channel based methods

$\square$ IBM
$\square$ Mays, Eric, Fred J. Damerau and Robert L. Mercer. 1991. Context based spelling correction. Information Processing and Management, 23(5), 517-522
$\square$ AT\&T Bell Labs
$\square$ Kernighan, Mark D., Kenneth W. Church, and William A. Gale. 1990. A spelling correction program based on a noisy channel model. Proceedings of COLING 1990, 205-210

