Evolutionary Cube Solver

Anurag Misra
Dept. of Computer Science and Engineering
Indian Institute of Science and Technology Kanpur

RUBIK'S CUBE

- Classic 3*3*3 Rubik's Cube invented in 1974 by Erno Rubik
- Highly complex puzzle
- 4.3 * 10^19 unique configurations
- Only 1 of these → "solved state"
- Smallest number of moves to solve ("God's Number") yet unknown
- Only few exact approaches exist
- Most (promising) based on group theory
- No valid evolutionary approach incorporating group theory until now

RUBIK’S CUBE

- Each face is referred to by its position (relative to users viewpoint)
- Common notation is F, R, U, B, L, D
- These also stand for a 90 degree clockwise turn
- Correspondingly Fi, Ri, Ui, Bi, Li, Di denote counter-clockwise 90-degree turn.
- Moreover, F2, R2, U2, B2, L2, D2, correspond to clockwise half turns

How to go about it?

Idea

- Take human strategies and incorporate them into an evolutionary approach
- Use group theoretical background to reduce complexity

Result

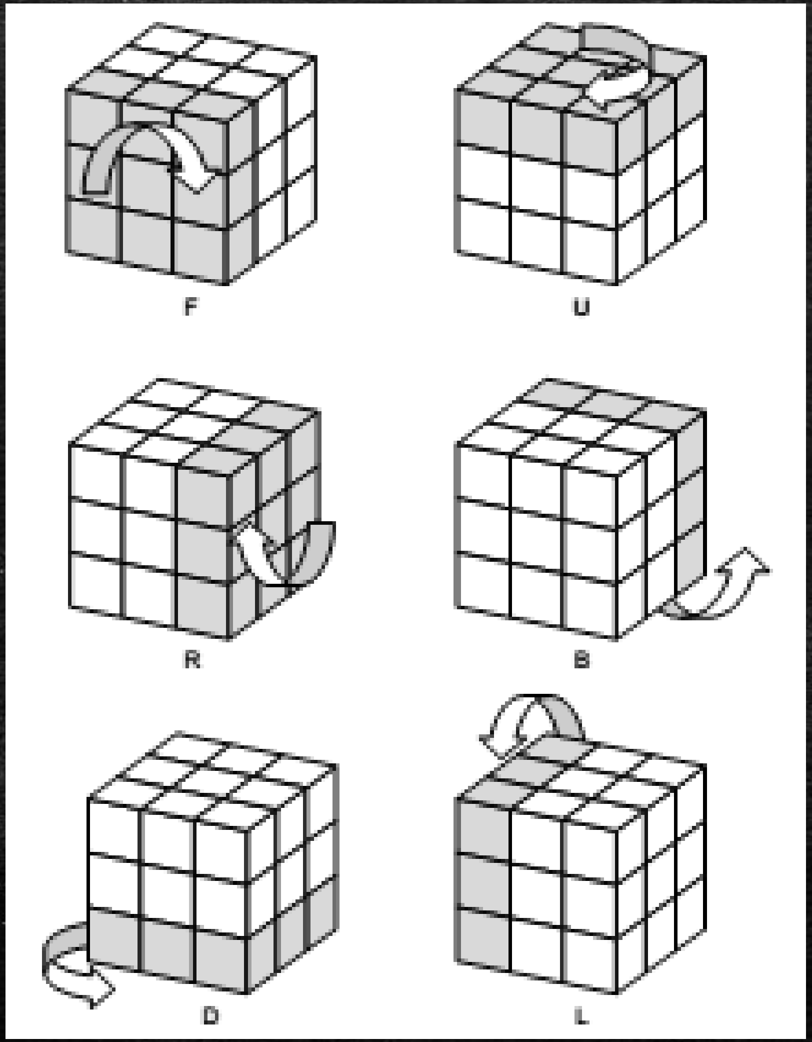
- A more powerful evolutionary algorithm adapting human strategies and incorporating exact methods
- Symbiotic Intelligence

Advantage

- No need of terabytes of pre-calculated lookup tables

- Study human strategies
- Use group theoretic background
- Evolve an algorithm

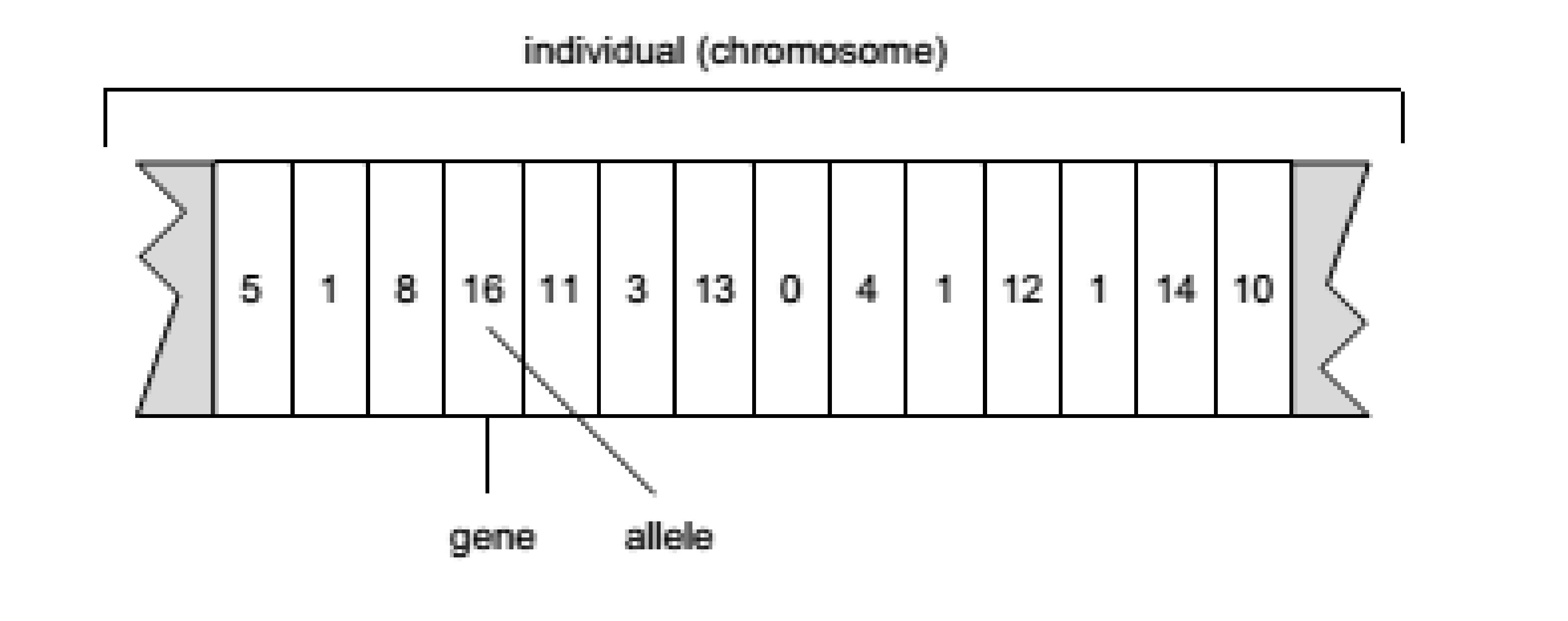
Human strategy based genetic optimizer



Human strategy based genetic optimizer

Clockwise quarter turns		Half turns		Counter-clockwise quarter turns	
F	0	F2	6	F'	12
U	1	U2	7	U'	13
R	2	R2	8	R'	14
B	3	B2	9	B'	15
D	4	D2	10	D'	16
L	5	L2	11	L'	17

Human strategy based genetic optimizer



EXISTING EXACT ALGORITHM: Thistlewaite Algorithm

- Developed by Morgan Thistlewaite in 1984
- Divides the problem of solving the cube into 4-subproblems

Definition

$$G_0 = < F, R, U, B, L, D > \supset$$
$$G_1 = < F, U, B, D, R2, L2 > \supset$$
$$G_2 = < U, D, R2, L2, F2, B2 > \supset$$
$$G_3 = < F2, R2, U2, B2, L2, D2 > \supset$$
$$G_4 = I$$

with $|G_0| > |G_1| > |G_2| > |G_3| > |G_4|$.

EXISTING EXACT ALGORITHM: Thistlewaite Algorithm

- Transition cube from Gi → Gi+1 only using moves from Gi
- Pre-calculated lookup-tables, solves in max. 52 moves

EXISTING EXACT ALGORITHM: Thistlewaite Algorithm

Definition

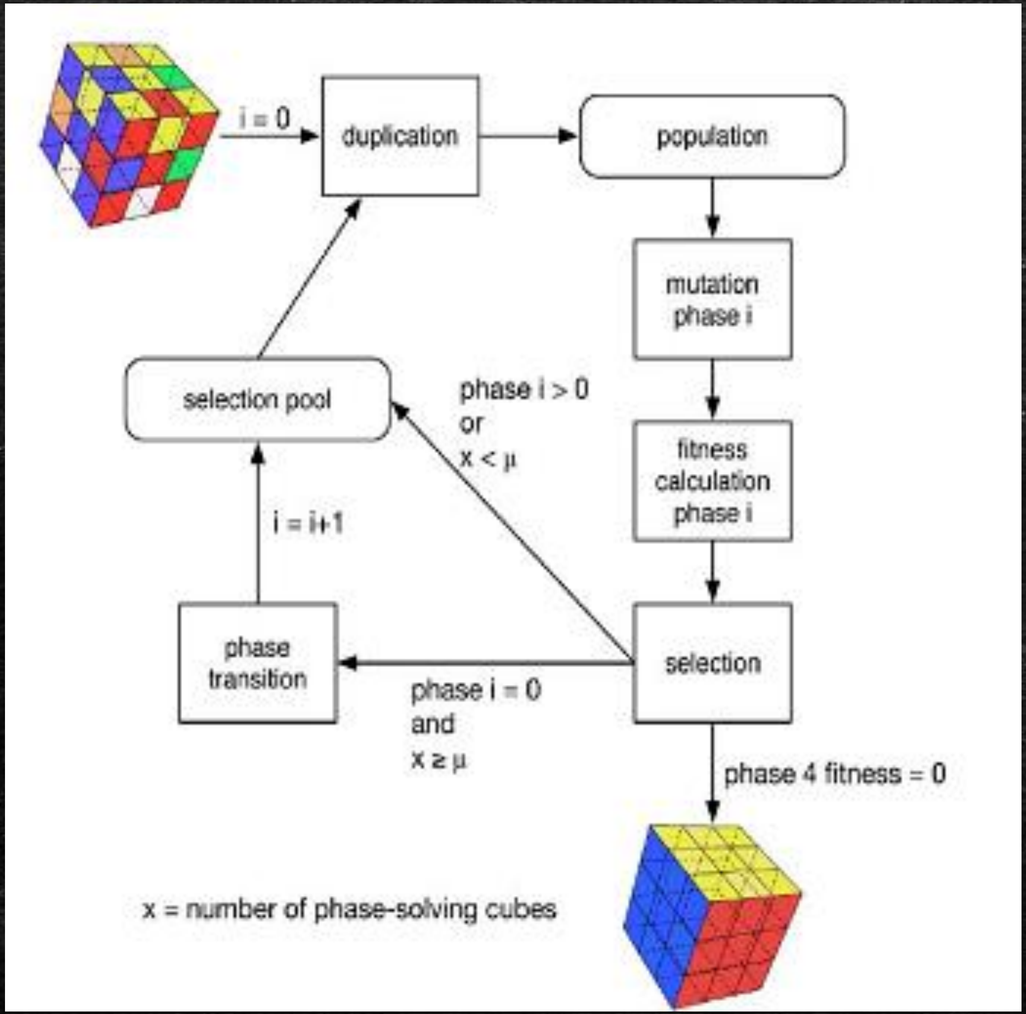
A subset $S \subseteq G$, is called a *generator* of G if any element of G can be written of a product of elements of S and their inverses. This is denoted by $G =< S >$.

- Thus G(c) = <F, R, U, B, L, D> (“Cube Group”) with |G(c)| = 4.3 * 10^19

EXISTING EXACT ALGORITHM: Thistlewaite Algorithm

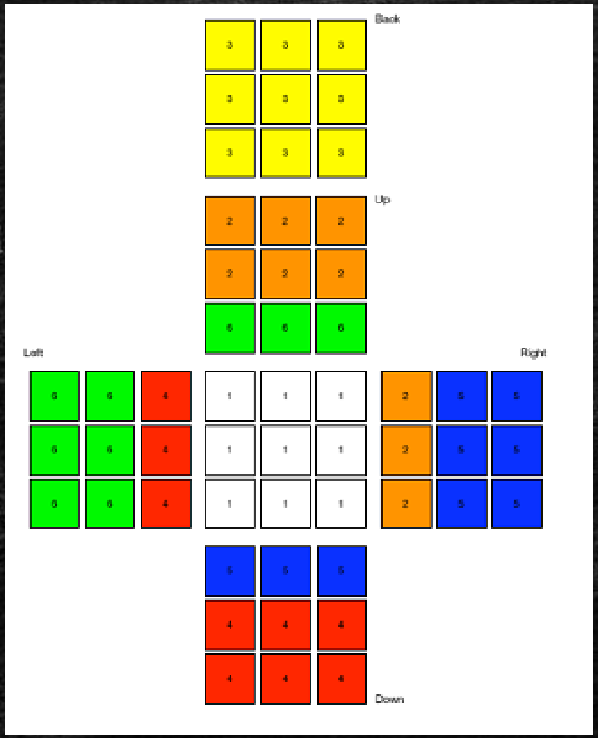
- G(0), |G(0)| = 4.3 * 10^19 — * no constraint
- G(1), |G(1)| = 2.11 * 10^16 — * orientation of edge cubies
- G(2), |G(2)| = 1.95 * 10^10 — * orientation of corner cubies
  transport of edge cubies to/from middle layer
- G(3), |G(3)| = 6.63 * 10^5 — * ……

State Complexity Reduction by Evolutionary Phase Transition



Evolution Strategy

Rubik's cube as an individual

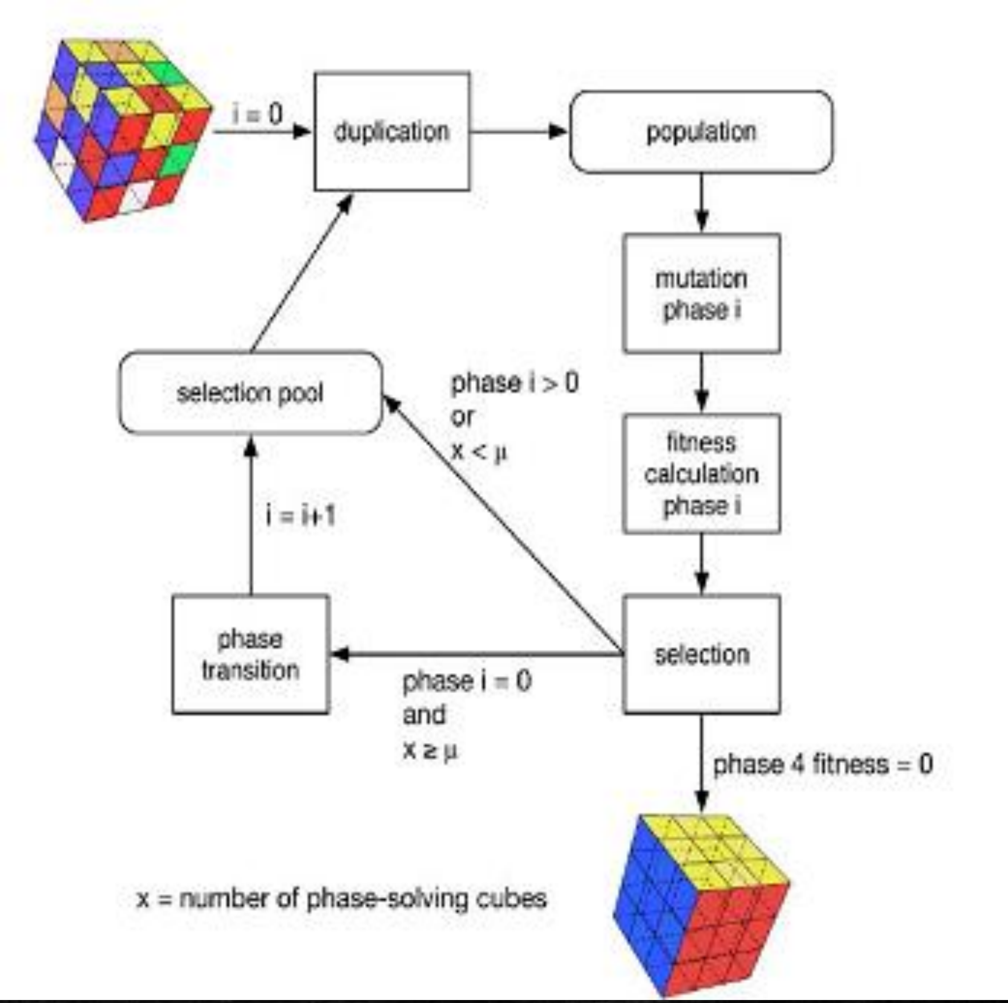


RUBIK's cube

- Represented using 6 2D matrices
- Can be mutated only by applying move sequences
- Remembers all mutations undergone as a sequence list
- Automatically removes abundant moves after each mutation
- Remembers optimized sequence only

State Complexity Reduction by Evolutionary Phase Transition

- Scrambled cube is duplicated λ times



Evolutionary Phase Transition

- Each phase has it's own fitness function, counting
  - Wrong oriented/positioned cubies according to group constraints
  - Length of the remembered sequence list
- Weights adjustable

Example G(0) → G(1):

phase(0) fitness = weight.(w) + c

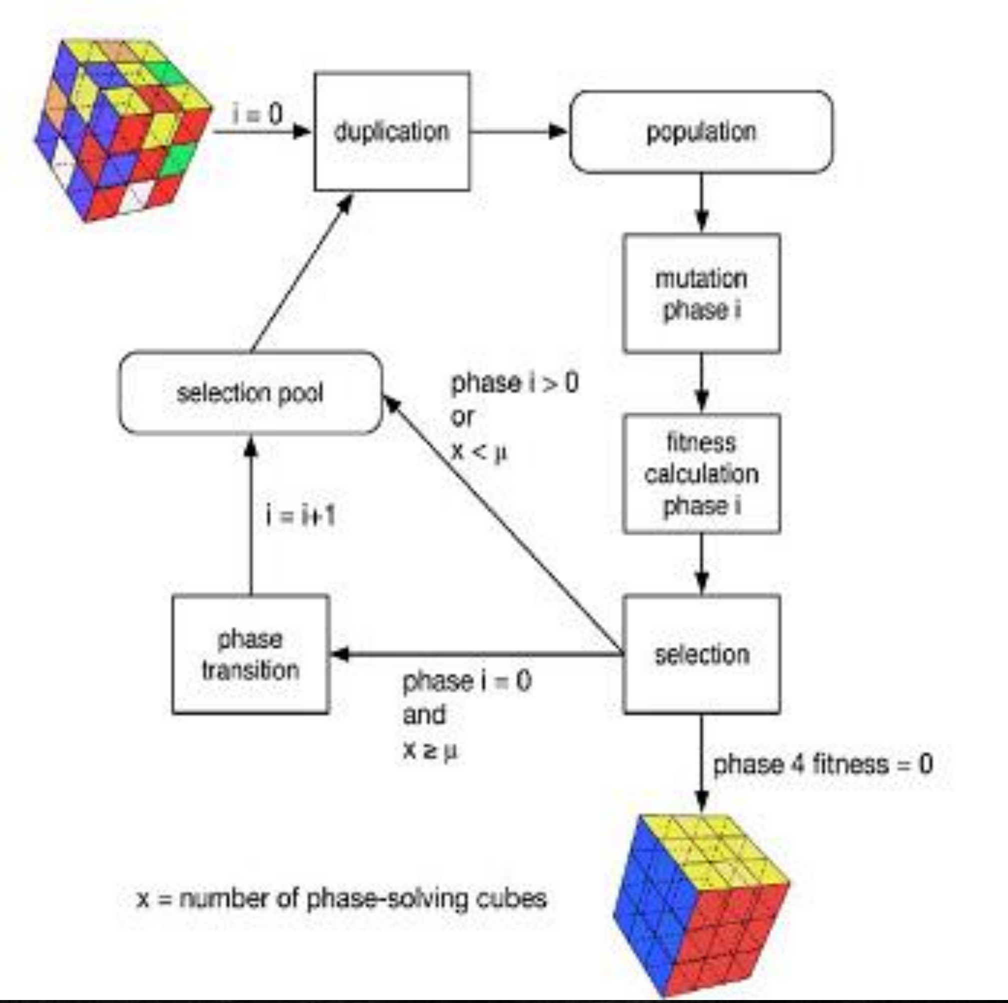
w: = number of wrong oriented edges

c: = length of the sequence list

G(i) constraints satisfied if phase(i) fitness = c

State Complexity Reduction by Evolutionary Phase Transition

- Scrambled cube is duplicated λ times
- Yields first population after the phase transition
- Process is repeated until phase-4 is solved
- Selection pool generated by choosing best μ individuals from current population



References

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QUESTIONS?