Rubik's Cube Solver using Thistlewaiste's ES Strategy "An Evolutionary Approach for Solving" "the Rubik-Cube Incorporating Exact Methods" by Nail El-Sourani, Sascha Hauke, and Markus Borschbach, University of Applied Sciences, Germany Advised by Dr. Amitabha Mukherjee, IIT Kanpur

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Abstract

The research paper "An Evolutionary Approach for Solving the Rubik-Cube Incorporating Exact Methods" by Nail El-Sourani, Sascha Hauke, and Markus Borschbach, University of Applied Sciences, Germany applies group theory and evolutionary strategy to solve Rubik's Cube. Though previously too there have been methods to solve the Rubik's Cube, like Thistlewaiste's and Kociemba's and Rokicki's The nobility with their approach is that it doesnot require terabytes of lookup tables unlike the former algorithms. However, the integrity and success of their method is still needs to tested and verified.

1. Introduction

Rubik's Cube since introduced in 70's has been the most intriguing puzzles in the history of mankind. It is a challenging task. The size of the solution space because possible number of configurations of the Rubik's cube and an attempt to optimize the solution, i.e. lowest possible number of moves and lowest calculation complexity makes it a very interesting optimization problem. The least possible number of moves to solve the Rubik's Cube, also known as the God's number is however yet unknown. All the algorithms so far have focused on decreasing the upper bound. All the approaches so far are strictly exact methods and require terabytes of pre-calculated lookup tables, reflected by current lowest upper bound of 22 moves(ROKICKI'S ALGORITHM).

Evolutionary or Genetic Algorithms have been applied in a large number of problems, like the Traveling Salesman Problem, and sometime deliver better superior solutions than the classical algorithms. This suggests the idea of applying it to solve the Rubik's Cube. All relevant methods so far involve partitioning the solution space into mathematical groups. Thistlewaiste's uses 4, while Kociemba's and Rokicki's uses 2 subgroups. This makes their approach precursor for developing Evolutionary Algorithms; interesting issue is to do it without look-up tables.

2. Thistlewaiste's Algorithm

Among of the one of primary exact methods, it was chosen to work further on developing an Evolutionary Algorithm for solving the Rubik's Cube. TWA divides the problem into four subproblems using the four nested groups: G0=¡F, R, U, B, L, D ¿, G1=¡ F,U,B,D,R2,L2¿, G2=¡ U,D,R2,L2,F2,B2¿, G3= ¡F2,R2,U2,B2,L2,D2¿, G4=I i.e. the Solved State. The functional principle is to put the cube into a state such that it can thence be solved only from moves in the sequences generated from G(i+1) group. The state is however achieved by using moves from the G(i) group. Every stage is simply a lookup table showing a transition sequence for each element in the current coset space Gi+1/Gi to the next one (i=i+1). These coset spaces are refinements of the earlier solution, limiting the possible configurations of the Rubik's cube in each state.

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The exact orders for each group are calculated as follows: $G_0, |G_0| = 4.33 * 10^{19}$represents the order of the Cube Group.

G_1: The first coset space G1/G0 contains all Cube states, where the edge orientation does not matter. This is due to the impossibility of flipping edge cubies when only using moves from G1. As there are 2^{11} possible edge orientations,$|G_1/G_0| = 2^{11} = 2048$

The order of$|G_1|$ is $|G_1||G_0||G_1/G_0| = 2.11 * 10^{16}$.

(2) G_2 Using only moves from G_2, no corner orientations can be altered (eliminating 3^7 states). Additionally, no edge cubies can be transported to or from the middle layer (eliminating 12!/(8!∗4!) states). The coset space G_2/G_1 thus

depicts a reduced puzzle of the order $|G_2/G_1| = 3^7 * 12!/(8! * 4!) = 1082565$ and $|G2||G1|/|G_2/G_1| = 1.95 * 10^{10}$
(4) G_3: Once in the coset space G_3/G_2, the Cube can be solved by only using moves from G_3, here the edge cubies in the L, R layers can not transfer to another layer (eliminating 8!/(4!*4!)*2 states) and corners are put into their correct orbits, eliminating 8!(4!*4!)*3 states). Thus,$|G_3/G_2| = (8!/(4! * 4!)) * 2 * 2 * 3 = 29400$ and $|G_3||G_2|/|G_3/G_2| = 6.63 * 10^5$.
G_4 as G_4 represents the solved state - obviously $|G_4| = 1$ which means there exist a mere $|G_3|$ possible states for which a solution needs to be calculated to transfer from G_4/G_3 to solved state. Most essential to TWA are the groups G_1, G_2, G_3 as G_0 simply describing the Cube Group G_c and G_4 the solved state. To further exemplify how the coset spaces simplify the Rubiks Cube puzzle the following may prove helpful.When looking at the constraints induced by $G_2/G_1/G_0$ carefully (combining the constraints induced by G_2/G_1 and G_1/G_2) it is essentially a Rubiks Cube with only 3 colors - counting two opposing colors as one. This representation can be reached for each distinct coset space by examining and applying its effect to the complete Rubiks Cube puzzle.While solving the Rubiks Cube in a divide and conquer manner, breaking it down into smaller problems (by generating groups and coset spaces) is effective,there exists one major flaw. Final results obtained by concatenating shortest subgroup solution do not necessarily lead to the shortest solution, globally.

3. The Thistlethwaite ES - An Evolution Strategy Based on the Thistlethwaites Algorithm

In the classic TWA the order of each subproblem get reduced from stage to stage. This algorithm present a 4-phase ES. Each phase here has a fitness function.

A scrambled Cube is duplicated times and the main loop is entered with a fitness function. Mutation sequences are generated using the group G_0 started using a fitness function $phase_0$. As soon as Cubes which solve $phase_0$ have been evolved, the phase transition begins. During phase transition, from those $phase_0$-solving Cubes, a random Cube is chosen and duplicated. This is repeated times and yields in the first population after the phase transition. Now the phase-counter is increased by one, and the main ES loop is entered again. This process is repeated until $phase_4$ is solved (i.e. $phase_5$ is reached), presenting a solution sequence to the originally scrambled Cube. In order to avoid the TWES getting stuck in local optima an upper bound for calculated generations is introduced. As soon as this upper bound is reached, the chain is terminated,

Fitness Function translating the TWA into an appropriate Fitness Function for an Evolutionary Algorithm essentially forces the design of four distinct sub-functions. As each subgroup of G_0 has different constraints, custom methods to satisfy these constraints are proposed.

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G_0 to G_1 To reach G_1 from any scrambled Cube, we have to orient all edge pieces right while ignoring their position. The fitness function for this phase simply increases the variable $phase_0$ by 2 for each wrong oriented edge. Furthermore, we add the number of moves that have already been applied to the particular individual in order to promote shorter solutions. Finally, we adjust the weight between w(number of wrong oriented edges) and c(number of moves applied to current Cube individual). This will be done similarly in all subsequent phases.

$phase_0 = 5 * (2w) + c$ With a total of 12 edges which can all have the wrong orientation this gives max2w= 24. The Cube has been successfully put into G_1 when $phase_0 = c$.Reaching G_1 is fairly easy to accomplish, thus making the weight-factor 5 a good choice.

G_1 to G_2 In order to fulfill G_2 the 8 corners have to be oriented correctly. Edges that belong in the middle layer get transferred there. Tests with the Thistleth-waite ES showed it somewhat problematic to do this in one step. Oftentimes,the algorithm would get stuck in local optima. To solve this, the process of transferring a Cube from G_1 to G_2 has been divided into two parts. First, edges that belong into the middle layer are transferred there. Second, the corners are oriented the right way. The first part is fairly easy and the fitness function is similar to that from $phase_0$ except for w(number of wrong positioned edges),i.e. edges that should be in the middle layer but are not. $phase_1 = 5(2w) + c$. In the second part, for each wrong positioned corner, 4 penalty points are as-signed as they are more complex to correct than edges. Obviously, in order to put the Cube from G_1 to G_2 both phases described here have to be fulfilled, which yields: $phase_2 = 10(4v) + phase_1$ where v represents the number of wrong oriented corners. The weighing factor is increased from 5 to 10 to promote a successful transformation into G_2 over a short sequence of moves.

G_2 to G_3 We now have to put the remaining 8 edges in their correct orbit. The same is done for the 8 corners which also need to be aligned the right way. Thus, the colors of two adjacent corners in one circuit have to match on two faces. In G_3 the Cube will only have opposite colors on each face. Let x (number of wrong colored facelets) and y (number of wrong aligned corners), then $phase_3 = 5(x + 2y) + c$.
An Evolutionary Approach for Solving the Rubiks Cube G_3 to $G_4(solved)$ The Cube can now be solved by only using half-turns. For the fitness function we simply count wrong colored facelets. Let z be the number of wrong colored facelets, then $phase_4 = 5z + c$.
To summarize, 5 different fitness functions are needed for the Thistlethwaite ES. $phase_i$ is solved if $phase_i = c$, i=0, ...,4 and with the properties of nested groups we can conclude, given the above, a solved Cube implies: $phase_i = c$. Fulfilling the above equation satisfies the constraints induced by the groups G0,...,G4, with the final fitness value c describing the final solution sequence length. The maximum sequence length (s) needed to transform the Cube from one subgroup to another is given by Thistlethwaite. Those lengths are 7,13,15,17 (the sum of which is 52, hence 52 Move Strategy).

4. Conclusion

Verification of the claims of the testbenchs and results given in the research paper is still under process. No result can therefore be quoted hencefar. We expect however, the algorithm delivers as promised. Work is currently under progress. Code would be uploaded on github, and link shared.

5. References

1. An Evolutionary Approach for Solving the Rubiks Cube Incorporating Exact Methods by Nail El-Sourani, Sascha Hauke, and Markus Borschbach, University of Applied Sciences,Faculty of Computer Science, Germany.
2. Borschbach, M., Grelle, C.: Empirical Benchmarks of a Genetic Algorithm Incorporating Human Strategies. Technical Report, University of Applied Sciences, Ber-gisch Gladbach (2009)

3. Boyzejko, W., Wodecki, M.: A Hybrid Evolutionary Algorithm for some Discrete Optimization Problems. In: Proceedings of the 5th International Conference on Intelligent Systems Design and Applications, pp. 326331. IEEE Computer Society,Washington (2005)
4. El-Sourani, N.: Design and Benchmark of different Evolutionary Approaches to Solve the Rubik's Cube as a Discrete Optimization Problem. Diploma Thesis, WWU Muenster, Germany (2009)
5. Florios, K., Mavrotas, G., Diakoulaki, D.: Solving Multiobjective, Multiconstraint Knapsack Problems Using Mathematical Programming and Evolutionary Algorithms. European Journal of Operational Research 203, 1421 (2009)
6. Frey, A., Singmaster, D.: Handbook of Cubic Math. Enslow, Hillside (1982)
7. Herdy, M., Patone, G.: Evolution Strategy in Action, 10 ES-Demonstrations. Technical Report, International Conference on Evolutionary Computation (1994)
8. Kociemba, H.: Cube Explorer,http://kociemba.org/Cube.htm
9. Muehlenbein, H., Mahnig, T.: FDA - A Scalable Evolutionary Algorithm for the Optimization of Additively Decomposed Functions. Evol. Comput. 7, 353376 (1999)
10. Reid, M.: Cube Lovers Mailing List, http://www.math.rwth-aachen.de/ Martin.Schoenert/Cube-Lovers/IndexbyAuthor.html
11. Rokicki, T.: Twenty-Five Moves Suffice for Rubiks Cube, http://Cubezzz.homelinux.org/drupal/?q=node/view/121
12. Rokicki, T.:http://www.newscientist.com/article/mg19926681.800-cracking-the-last-mystery-of-the-rubiks-Cube.html
13. Singmaster, D.: Notes on Rubiks Magic Cube. Enslow, Hillside (1981)
14. Thistlethwaite, M.B.: The 45-52 Move Strategy. London CL VIII (1981)
15. Zitzler, E.: Evolutionary Algorithms for Multiobjective Optimization: Methods andApplications. Penn State (1999)