**Topological Data Analysis (TDA)**

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### Introduction

**Topology** is the branch of mathematics that deals with the study of shape of data.

**TDA:**

- Data Points → Geometric object → Topological summary

There are three key ideas of topology that make extraction of patterns via shape possible:

1. It studies shapes in a coordinate-free way.
2. It studies the properties of shapes that are invariant under 'small deformations'.
3. The third key idea is that of compressed representation of shapes. Betti numbers are used to distinguish topological spaces based on the connectivity of n-dimensional simplicial complexes.

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### Barcode Analysis of Heart Disease Dataset

**Barcode:**

- Each horizontal bar represents the birth-death of a separate homology class.
- The i-th Betti number at any given parameter value is the number of bars.

**Observations**

The graph shows variation of the structure with different values of the threshold parameter t.

By observing the barcode of dimension 0 we can infer that a single line persists in our data set.

This implies that b₀ is 1, meaning that there is a singly connected component.

From the graph of dimension 1, we can see that some structures come up and decay. There is no persistent homology in this structure in 1 dimension.

Thus b₁ for our data is 0.

These numbers imply that there is no persistent circle in our data.

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### MAPPER

Point Cloud → Filter Function → Binning → Clusters

**Filter Function**

- L-infinity

**Point Cloud Data**

- N-dimensional (Can be pre-processed as well)

**Filter Function**

- \( f(x, y, z) \rightarrow x \)

**Binning**

- \( f'(a, b) \)

- Put data into overlapping bins

**Cluster each bin and create a network**

- Vertex = cluster of bin
- Edge = non-empty intersection between clusters

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### References

2. Carlsson, G. et al. (2013) 'Extracting insights from the shape of complex data using topology',

**Dataset:** Heart Disease Data Set, Hungarian Institute of Cardiology, Budapest; Andras Janosi, M.D.