CS365: Artificial Intelligence

Quiz: Logic Name: ______________________

Question 1. **(Formalism):** [5] Construct a turth-table and show that the rule of Material Equivalence is a tautology.

Solution:

p	q	$p \Rightarrow q$	$q \Rightarrow p$	$p \equiv q$	$(p \Rightarrow q) \wedge (q \Rightarrow p)$	$(p \equiv q) \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$
T	T	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	F	F	F	T
F	F	T	T	T	T	T

Since $(p \equiv q) \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$ is true for all truth assignments of p and q, it is a tautology.

Question 2. **(Propositional Logic):** [10]
Prove the following using either resolution or traditional logic, using these propositions:
S : I study; G: I get good grades; E: I enjoy.

1. If I study I make good grades.
2. If I do not study I enjoy.
   ∴ either I make good grades or I enjoy.

Solution:

Traditional logic proof (one of many possible solutions):

1. $S \Rightarrow G$
2. $\neg S \Rightarrow E$
   $\therefore G \vee E$
3. $\neg E \Rightarrow S$ (2; transp.+D.N.)
4. $\neg E \Rightarrow G$ (3,1; H.S.)
5. $E \vee G$ (4; M.I.+D.N.)
6. $G \vee E$ (5; Comm.) □

Resolution Refutation Proof:
(statements 1 to 4 are the Clause Form of the given statements; 3,4 are the Goal negation)

1. $\neg S \vee G$ [1]
2. $S \vee E$ [2]
3. $\neg G$ [G.N. part a]
4. $\neg E$ [G.N. part b]
5. $E \vee G$ (1,2)
6. G (5,3)
7. NIL (6,3) □

Question 3. **(First-Order Logic):** [10]
Express the following in FOL and construct a proof using resolution refutation:

1. Everyone has a parent
2. For any persons x, y, and z, if z is y's parent and y is x's parent, then z is the grandparent of x.
3. Therefore, everyone has a grandparent.

Solution:

Translation to F.O.L. with predicates P(x,y) [x is parent of y] and G(x,y):

1. $\forall x \; \exists y \; P(y,x)$
2. $\forall x \; \forall y \; \forall z \; [P(y,x) \wedge P(z,y) \Rightarrow G(y,z)]$
3. $\therefore \forall x \; \exists y \; G(y,x)$

In resolution refutation, getting the **clause form** is the most critical step.

clause 1:
the clause says that "for every x, there is someone (y) who is the parent of x."
while removing $\exists y$, we must use a new *skolem function*: P(f(x),x). This notation is simply saying that the y for which $P(y,x)$ is true, may be different for [has some dependence on] each x.

Common error: replacing $\exists y$ by a constant – e.g. p(A,x) – is saying that A is the parent for everyone. clearly not what is intended.

clause 2:
after removing quantifiers and implication, we have:

$\neg[P(y_2,x_2) \wedge P(z_2,y_2)] \vee G(z_2,x_2)$

which says that:

either (y_2 is not the parent of x_2 and the child of z_2),
or (z_2 is the grandparent of x_2).

and after de Morgan's, we get:

$\neg P(y_2,x_2) \vee \neg P(z_2,y_2) \vee G(z_2,x_2)$

which says that:

either (y_2 is not the parent of x_2)
or (y_2 is not the child of z_2),
or (z_2 is the grandparent of x_2).

Goal negation:
$\exists x \; \forall y \; \neg \; G(y,x)$; (i.e. there is an x who has no grandparent)
which becomes $\neg \; G(y_3,A)$; where A is a skolem constant.

Common error: the negation must be applied before removing quantifiers. otherwise, you'll get $\neg G(g(x),x)$. This says "everyone has no grandparent", which is much stronger than "someone has no grandparent".

In the derivation below, clauses 1,2 are from the statements and Clause 3 from the Goal Negation.

Proof using Resolution Refutation:

1. $P(f(x_1),x_1)$ [1]
2. $\neg P(y_2,x_2) \vee \neg P(z_2,y_2) \vee G(z_2,x_2)$ [2]
3. $\neg \; G(y_3,A)$ [GN]
4. $\neg P(y_2,A) \vee \neg P(z_2,y_2)$ [2,3; $s = \{x_2/A, y_3/z_2\}$]
5. $\neg P(z_2,f(A))$ [4,1; $s = \{x_1/A, y_2/f(A)\}$]
6. NIL [5,1; $s = \{x_1/f(A), z_2/f(f(A))\ \}$]