Advances in matrix manifolds for computer vision

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This paper presents an overview of various matrix manifolds that are commonly used in computer vision applications. It covers the following manifolds- Lie groups, Stiefel manifolds, Grassmann manifolds and Riemannian manifolds. A manifold of dimension n is a topological space that near each point resembles n-dimensional Euclidean space. More precisely, each point of an n-dimensional manifold has a neighbourhood that is homeomorphic to the Euclidean space of dimension n. Eg:- Lines and circles are one-dimensional manifolds, but not figure eights.

Lie group A Lie group is a set G with two structures: G is a group and G is a (smooth, real) manifold. These structures agree in the following sense: multiplication and inversion are smooth maps.

Stiefel manifold The Stiefel manifold of orthonormal -frames in R^n is the collection of vectors (v1,v2 ...,vn) where v_i is in R^n for all i, and the k-tuple (v1, v2...,vk) is orthonormal.

Grassmann manifolds is a certain collection of vector subspaces of a vector space. In particular, g_{nk} is the Grassmann manifold of k-dimensional subspaces of the vector space.

Riemannian manifolds- In the Riemannian framework, the tangent space TxM at each point x of a manifold M is endowed with a smooth inner product <·, ·>x.

In computer vision applications we relate the data to some manifold through various constraints that are imposed on it. In some applications, the characteristics of visual patterns may be described by some transformation whose state transition is governed by a group action. We may then exploit the parameter in the transformation space. This manifold approach has been viewed as important for understating various aspects of vision like face recognition, activity recognition and clustering.

Face Recognition Researcher have exploited discriminant analysis for face recognition using a learned projection to mapimages to a lower dimensional space where the projection was viewed as an element on a Grassmann manifold. The gradient flow on the Grassmann manifold was approximated and the locally optimal projection was found using gradient directed search.

Activity Recognition Human activity and action recognition is the recent field that has emerged that uses matrix manifolds. modeled human shapes on a shape manifold where sequences were first normalized using Dynamic Time Warping (DTW). A sequence of shape changes was then extracted from the tangent space and an ARMA model was exploited to learn the dynamics of the human movement. The space spanned by the parameters of the linear dynamic system was identified as an element on a Grassmann manifold.

Clustering Grouping similar patterns into a cluster or computing data centroids is an important process in unsupervised learning. Researchers have incorporated Lie groups with the Mean-Shift (MS) algorithm for 3D motion estimation. The motion parameters represented on a Lie group were estimated via mode finding on the sampled distribution. The kernel density function was expressed using an intrinsic distance through the BCH formula. The modes of the underlying distribution were then computed iteratively.

geometry may be the most fundamental basis in pattern analysis. Matrix manifolds provide a natural way to characterize some visual objects. There are many emerging computer vision applications on the horizon. It can be advantageous to exploit the underlying geometry of the data. There has been a recent interest in associating matrix manifolds with statistical learning theory. Since computer vision is intimately related to machine learning, there may be potential impacts of investigating this area.

References

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