Solving the 8-Puzzle using
A* Heuristic Search

- **Admissible Heuristics**

  An admissible heuristic never overestimates the cost of reaching the goal. Using an admissible heuristic will always result in an optimal solution.

- **Non-Admissible Heuristics**

  A non-admissible heuristic may overestimate the cost of reaching the goal. It may or may not result in an optimal solution. However, the advantage is that sometimes, a non-admissible heuristic expands much fewer nodes. Thus, the total cost (= search cost + path cost) may actually be lower than an optimal solution using an admissible heuristic.

Admissible Heuristics for the 8-puzzle

7	2	4
5	*	6
8	3	1

Initial State

*	1	2
3	4	5
6	7	8

Final State

h1 : Number of misplaced tiles

In the above figure, all the tiles are out of position, hence for this state, ***h1 = 8.***

h1 is an admissible heuristic, since it is clear that every tile that is out of position must be moved at least once.

Admissible Heuristics for the 8-puzzle

7	2	4
5	*	6
8	3	1

Initial State

*	1	2
3	4	5
6	7	8

Final State

h2 : Sum of Eucledian distances of the tiles from their goal positions

In the given figure, all the tiles are out of position, hence for this state,

h2 = sqrt(5) + 1 + sqrt(2) + sqrt(2) + 2 + sqrt(5) + sqrt(5) + 2 = 14.53.

h2 is an admissible heuristic, since in every move, one tile can only move closer to its goal by one step and the eucledian distance is never greater than the number of steps required to move a tile to its goal position.

Admissible Heuristics for the 8-puzzle

7	2	4
5	*	6
8	3	1

Initial State

*	1	2
3	4	5
6	7	8

Final State

h3 : Sum of Manhattan distances of the tiles from their goal positions

In the given figure, all the tiles are out of position, hence for this state,
h3 = 3 + 1 + 2 + 2 + 2 + 3 + 3 + 2 = 18.
h3 is an admissible heuristic, since in every move, one tile can only move closer to its goal by one step.

Admissible Heuristics for the 8-puzzle

7	2	4
5	*	6
8	3	1

Initial State

*	1	2
3	4	5
6	7	8

Final State

h4 : Number of tiles out of row + Number of tiles out of column

In the given figure, all the tiles are out of position, hence for this state,
h4 = 5 (out of row) + 8 (out of column) = 13.
h4 is an admissible heuristic, since every tile that is out of column or out of row must be moved at least once and every tile that is both out of column and out of row must be moved at least twice.

Admissible Heuristics for the 8-puzzle

Some other admissible heuristics

- n-Max Swap

Assume you can swap any tile with the 'space'. Use the cost of the optimal solution to this problem as a heuristic for the 8-puzzle.

- n-Swap

Represent the 'space' as a tile and assume you can swap any two tiles. Use the cost of the optimal solution to this problem as a heuristic for the 8-puzzle.

Heuristics of this kind, which involve performing a search on a "relaxed" form of the problem (a method to invent admissible heuristic functions) will be covered in the second part of this presentation.

A Non-Admissible Heuristic for the 8-puzzle

Nilsson's Sequence Score

$$h(n) = P(n) + 3\,S(n)$$

P(n) : Sum of Manhattan distances of each tile from its proper position
S(n) : A sequence score obtained by checking around the non-central squares in turn, allotting 2 for every tile not followed by its proper successor and 0 for every other tile, except that a piece in the center scores 1

The goal state looks like this :

1	2	3
8	*	4
7	6	5

Admissible v/s Non-Admissible Heuristics

When solving a problem, one can have two kinds of objectives in mind :

- Minimize the path cost of the solution (i.e. find the optimal solution)
- Minimze the time taken to find the solution

Ofcourse, in most cases, both the objectives are important. A balance has to be struck between the two, and that is where the heuristic comes in.

An admissible heuristic is optimal, it will always find the cheapest path solution.

On the other hand, a non-admissible heuristic is not optimal, it may result in a suboptimal solution, but may do so in a much shorter time than that taken by an admissible heuristic.

Experimental results show that the Nilsson Sequence Score heuristic finds a solution to the 8-puzzle much faster than all the admissible heuristics.

Comparison of Heuristics

The quality of heuristics can be characterized on the basis of the **effective branching factor b***.
If the total number of nodes generated by A* for a problem is N, and the solution depth is d, then:

$$N = b^* + (b^*)^2 + (b^*)^3 + (b^*)^4 + \ldots.. + (b^*)^d$$

Experimental measurements of b* on a small set of problems can provide a good guide to the heuristic's overall usefullness.

A well designed heuristic would have a value of b* close to 1.

To compare the admissible heuristics mentioned earlier (h1 to h4), one can generate a large number of initial states for the 8-puzzle and solve each one using all 4 heuristics. The number of nodes expanded and depth of solution can be recorded and b values tabulated. Such a procedure would accurately reflect the relative quality of the heuristics.*

Comparison of Heuristics – Heuristic Accuracy

When can we say that heuristic a is *always* better than heuristic b?

If, from the definitions of the two admissible heuristics, a(n) > b(n) for all n, then we can say that a is a better heuristic than b, since it generates fewer nodes in the search tree. Another way to see this is that a(n) is a closer lower-bound to the actual path-cost to reach the goal, i.e., it is more *accurate*.

Among the admissible heuristics mentioned earlier (h1 to h4), we can see by the definition of the heuristics that :

h3 > h2 > h1 and h4 > h1

Thus, theoretically, the number of nodes generated or the effective branching factor in an A* search using these heuristics should be in the sam order. This can be verified by conducting an experiment of the kind mentioned in the previous slide.

Thus, among the admissible heuristics, Manhattan Distance is the most efficient.