Skylines: Databases' Answer to Multiple Preferences

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- Any booking site will show you hundreds of flights
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  - Not clear which one to choose
- Leave the choice to user by letting her examine both

SKYLINE QUERIES

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  - Flight C with more cost and less amenities than flight A
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Skyline Queries

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  - Flight C with more cost and less amenities than flight A
  - Flight C will *never* be preferred over flight A
- Skylines are those objects that are not dominated by some other object for all preferences
- Also known as Pareto optimal curve or maximal vector problem

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- The skyline query [BKS01] returns the set of objects $S \subseteq D$ such that for all objects $O_p \in S$ and all objects $O_q \notin S$,

$$O_p \in S \iff \nexists O_q \in D,\ O_q \succ O_p$$
$$O_q \notin S \iff \exists O_p \in S,\ O_p \succ O_q$$

Skyline Syntax in SQL

- PostgreSQL has adopted skyline keyword in SQL [Ede09]

```
SELECT fno, dep, arr, cost, rtg, amn
FROM flights
WHERE src = ``Kolkata'' and dest = ``Agartala'' and
SKYLINE of cost min, rtg max, amn max
```

Preference Functions

- If object O_p dominates object O_q, then for all monotone preference functions, O_p is better than O_q
- Moreover, for every skyline object O_p, there exists a monotone preference function f such that O_p optimizes f
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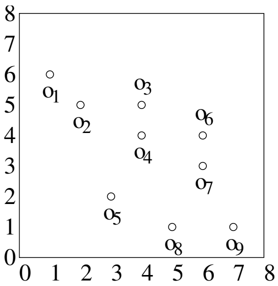
- This is why this is much more complex and interesting than a single optimization function
- There is no ranking or ordering among skyline objects

Example



- Assume preference functions are less ($<$)

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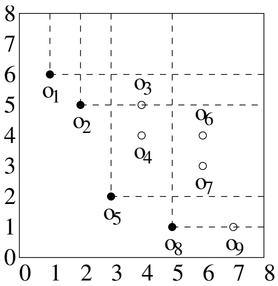

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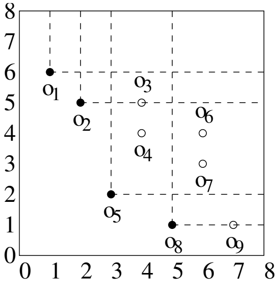

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- - - lines show the region dominated by the corresponding object
- Skyline objects are O_1, O_2, O_5, O_8

Naïve Algorithm

- Compare each object with every other object
- An object is a skyline only if no object dominates it

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- $O(n^2)$ for n objects

Block-nested-loop (BNL) algorithm

- First skyline algorithm [BKS01]
- Maintain a window of objects not yet dominated
- Check every new object against the window
  1. If it is dominated, prune it
  2. If it dominates some other object(s) in the window, prune those object(s)
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- Finally, window contains *only* the skyline objects

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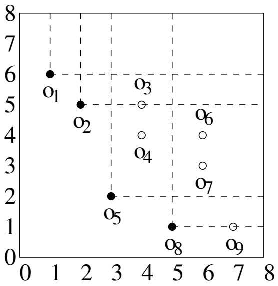
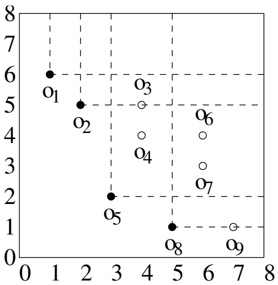


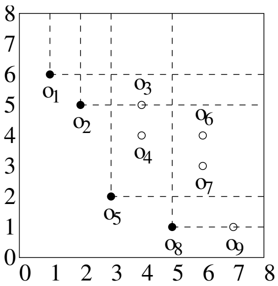
Window: Φ

Example



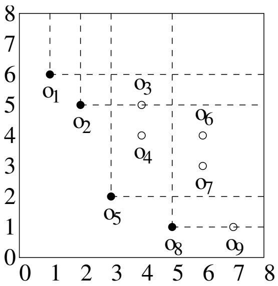
Window: O_1

Example



Window: O_1, O_2

Example



Window: O_1, O_2
Pruned: O_3

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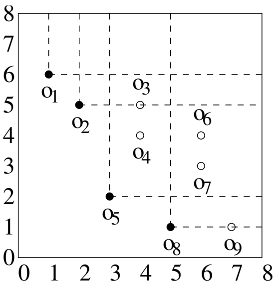
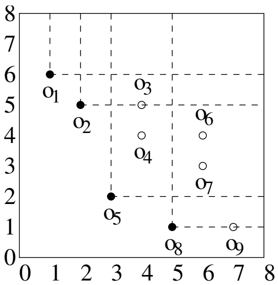


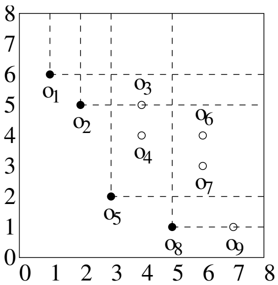
Window: O_1, O_2, O_4
Pruned: O_3

Example



Window: O_1, O_2, O_5
Pruned: O_3, O_4

Example



Window: O_1, O_2, O_5
Pruned: O_3, O_4, O_6

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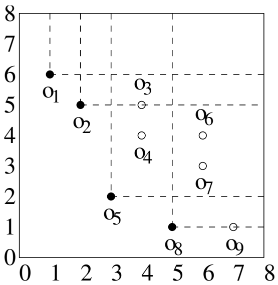

Window: O_1, O_2, O_5
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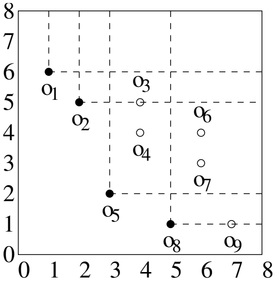

Window: O_1, O_2, O_5, O_8
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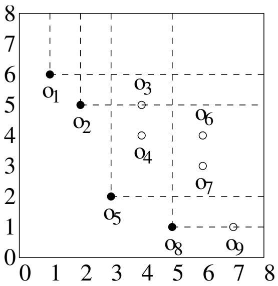

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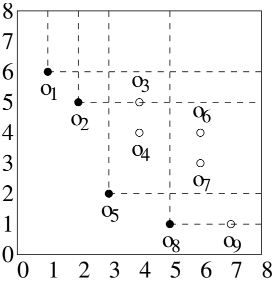

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Skylines: O_1, O_2, O_5, O_8

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- Every non-skyline object is dominated by at least one skyline object
  - Uses transitivity relationship of object dominance

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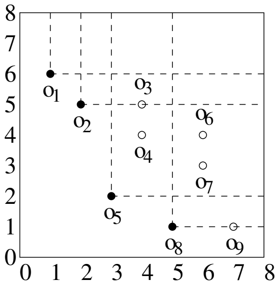
  - $A \succ B$ or A and B do not dominate each other
  - $B \not\succ A$ as then $E(B) < E(A)$

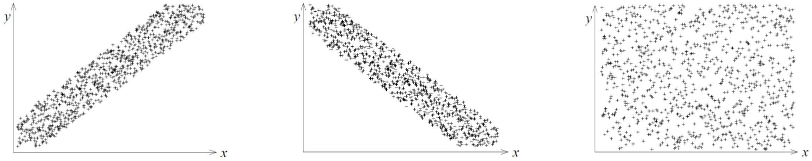
Example



Object	Value	Entropy
O_1	(1,6)	2.6390
O_2	(2,5)	2.8903
O_3	(4,5)	3.4011
O_4	(4,4)	3.2188
O_5	(3,2)	2.4848
O_6	(6,4)	3.5553
O_7	(6,3)	3.3321
O_8	(5,1)	2.4848
O_9	(7,1)	2.7725

- Sorting produces a much better scanning order
- Here, it is O_5, O_8, O_1, O_9, O_2, O_4, O_7, O_3, O_6

Number of Skyline Objects



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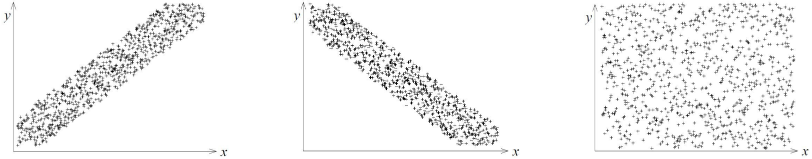

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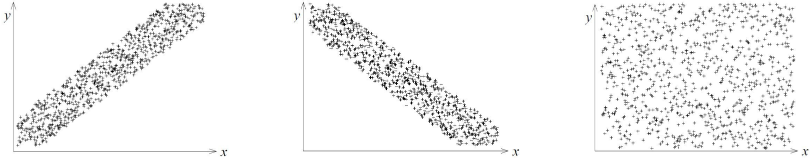

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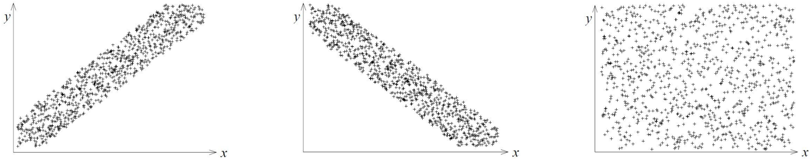

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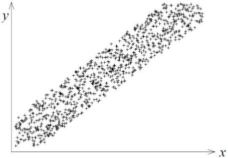

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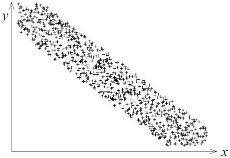
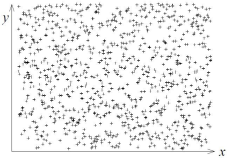


Anti-correlated



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- For *independent* data, skyline cardinality is

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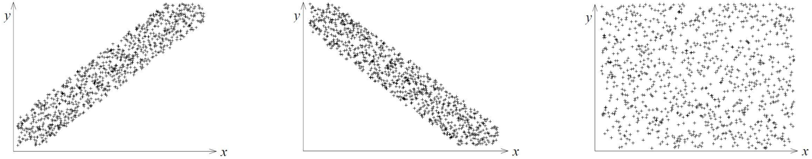

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  - Estimated skyline cardinality is

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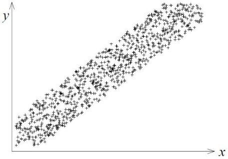

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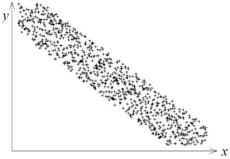

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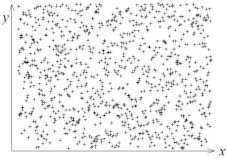

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- For *independent* data, skyline cardinality is *medium*
  - Estimated skyline cardinality is $\ln^{d-1} N/(d-1)!$ [God04]

Skylines in High Dimensions

- For large high-dimensional datasets, number of skyline objects quickly becomes unmanageable

N	d				
	1	2	3	5	10
10^3	1	7	24	95	99
10^4	1	9	42	300	1314
10^5	1	12	66	732	9793
10^6	1	14	95	1518	50529
10^7	1	16	130	2812	202330

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- In high dimensions, it is rare that an object will dominate another in all skyline attributes
- It does not make sense to even show 300 flights for a database of 10,000 flights and 5 preference functions
- How to handle it?

k-Dominant Skylines

- Relax dominance to k-dominance [CJT+06]
- An object O_p k-dominates another object O_q if it is preferred in k of the d skyline attributes, i.e.,

$$O_p \succ_k O_q \iff \exists I, |I| = k, \quad \forall i \in I, O_{pi} \succeq O_{qi}, \quad \exists j \in I, O_{pj} \succ O_{qj}$$

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- A particular k-dominant skyline set may even be empty

A Two-Pass Algorithm for k-Dominant Skylines

- Maintain window W of k-dominant objects
- For every object $p \in D$, check against all objects $q \in W$
- If $p \succ_k q$, remove q from W
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- It may happen that $\exists r \in D \succ_k p$ but $\exists q \in W \succ_k r$ and, therefore, $r \notin W$ and r was not compared against p
- r must come earlier than p

A Two-Pass Algorithm for k-Dominant Skylines

- Maintain window W of k-dominant objects
- For every object $p \in D$, check against all objects $q \in W$
- If $p \succ_k q$, remove q from W
- If $q \succ_k p$, prune p
- Else, add p to W
- This first pass is not enough
- It may happen that $\exists r \in D \succ_k p$ but $\exists q \in W \succ_k r$ and, therefore, $r \notin W$ and r was not compared against p
- r must come earlier than p
- So, in second pass, check all $p \in W$ against all $r \notin W$ that came *earlier* in first pass

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- Naïve way is to join and then perform skyline query
- Skyline can be computed before join
  - If $u_1 \succ v_1 \in R_1$ and $u_2 \succ v_2 \in R_2$, then $u_1 \bowtie u_2 \succ v_1 \bowtie v_2 \in R$, and therefore, v_1 and v_2 need not be joined
  - Non-skyline sets need not be joined
  - Join of a skyline tuple is surely a skyline in R

Skylines over Aggregate Attributes

- Skyline preferences for joined relations are more often on attributes that have been aggregated from the base relations
- For example, a user would prefer less total cost of the two flights and not the individual costs
- However, total cost is only available *after* the join
- Aggregate skyline join queries [BT10]

Aggregate Skyline Join Queries (ASJQ)

- j join attributes; m_1, m_2 local attributes; n aggregate attributes
  - Join attributes: R_1.dest $= R_2$.src, R_1.time of arrival $< R_2$.time of departure
  - Local attributes: rating, amenities
  - Aggregate attributes: R.duration = R_1.duration + R_2.duration, R.cost = R_1.cost + R_2.cost

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- $A = \{h_{a_1}, \dots, h_{a_j}, \quad l_{a_1}, \dots, l_{a_{m_1}}, \quad g_{a_1}, \dots, g_{a_n}\}$
- $B = \{h_{b_1}, \dots, h_{b_j}, \quad l_{b_1}, \dots, l_{b_{m_2}}, \quad g_{b_1}, \dots, g_{b_n}\}$
- $J = \{h_{a_1} \bowtie h_{b_1}, \dots, h_{a_j} \bowtie h_{b_j}, \quad l_{a_1}, \dots, l_{a_{m_1}}, l_{b_1}, \dots, l_{b_{m_2}}, \quad g_{a_1} \oplus g_{b_1}, \dots, g_{a_n} \oplus g_{b_n}\}$
- Skyline is over preferences for $m_1 + m_2 + n$ attributes

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- Skyline is over preferences for $m_1 + m_2 + n$ attributes
- Naïve is too costly

Example

fno	dep	Join (H)		Aggregate (G)		Local (L)	
		arr	dst	duration	cost	amn	rtg
11	06:30	08:40	C	2h 10m	162	5	4
12	07:00	09:00	E	2h 00m	166	4	5
14	08:05	10:00	E	1h 55m	140	3	4
15	09:50	10:40	C	1h 40m	270	3	2
13	12:00	13:50	C	1h 50m	173	4	3
16	16:00	17:30	D	1h 30m	230	3	3
17	17:00	20:20	C	3h 20m	183	4	3

(a) Flights from city A (FlightsA)

fno	Join (H)		arr	Aggregate (G)		Local (L)	
	src	dep		duration	cost	amn	rtg
21	C	09:50	12:00	2h 10m	162	5	4
26	C	16:00	18:49	2h 49m	160	2	3
23	C	16:00	18:45	2h 45m	160	4	4
25	D	16:00	17:49	1h 49m	220	3	4
22	D	17:00	19:00	2h 00m	166	4	5
27	E	20:00	21:46	1h 46m	200	3	3
24	E	20:00	21:30	1h 30m	160	4	3

(b) Flights to city B (FlightsB)

f1.fno	f2.fno	f1.dst	f2.src	f1.arr	f2.dep	f1.amn	f2.amn	f1.rtg	f2.rtg	cost	duration	Skyline
11	21	C	C	08:40	09:50	5	5	4	4	324	4h 20m	Yes
11	23	C	C	08:40	16:00	5	4	4	4	322	4h 55m	Yes
13	23	C	C	13:50	16:00	4	4	3	4	333	4h 35m	No
15	23	C	C	10:40	16:00	3	4	2	4	430	4h 25m	No
12	24	E	E	09:00	20:00	4	4	5	3	326	3h 30m	Yes
14	24	E	E	10:00	20:00	3	4	4	3	300	3h 25m	Yes

(c) Part of the joined relation (FlightsA $\bowtie$ FlightsB)

SQL Syntax for ASJQ

- It is easy to incorporate ASJQ into existing SQL syntax

```
SELECT f1.fno, f2.fno,
  f1.dst, f2.src, f1.arr, f2.dep,
  f1.rtg, f2.rtg, f1.amn, f2.amn,
  cost as f1.cost + f2.cost,
  dur as f1.dur + f2.dur
FROM FlightsA as f1, flightsB as f2
WHERE f1.dst = f2.src and f1.arr < f2.dep and
SKYLINE of cost min, dur min,
  f1.rtg max, f2.rtg max, f1.amn max, f2.amn max
```

Full and Local Skylines

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Break-up using Full and Local Skylines

- Each relation R is broken up in the following way
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$$R = R_f \cup R_{f'}$$

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- Therefore,

$$R = R_f \cup R_{f'} = (R_{fl} \cup R_{fl'}) \cup R_{f'}$$

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  - Consider $t' = u \bowtie v'$ where $u \in A_{fl}$ and $v' \in B_{fl'}$
  - $\nexists u' \in A \succ_l u \implies \nexists u' \in A \succ_f u$
  - $\exists v \in B_{fl} \succ_l v'$ *only* in local as $v \in B_f$
  - So, $\nexists v \in B \succ_f v'$
  - Hence, the aggregate attributes of t' can never be dominated (monotone property)
  - Thus, t' is a skyline

State After using the Two Theorems

Set	Skyline	Target
$A_{fl} \bowtie B_{fl}$	yes	-
$A_{fl} \bowtie B_{fl'}$	yes	-
$A_{fl} \bowtie B_{f'}$	no	-
$A_{fl'} \bowtie B_{fl}$	yes	-
$A_{fl'} \bowtie B_{fl'}$	-	$A_f \bowtie B_f$
$A_{fl'} \bowtie B_{f'}$	no	-
$A_{f'} \bowtie B_{fl}$	no	-
$A_{f'} \bowtie B_{fl'}$	no	-
$A_{f'} \bowtie B_{f'}$	no	-

- Only the set $A_{fl'} \bowtie B_{fl'}$ needs to be checked
- It need to be checked only against $A_f \bowtie B_f$ and not the entire $A \bowtie B$

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  - More in context of skycube
- Uncertain databases
  - An object is a skyline with only some non-zero probability
  - This is because the dominator may not exist

Conclusions

- Skylines provide a nice way of handling multiple preferences
- Algorithms need to be practical
- Some nice theoretical issues as well
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QUESTIONS?
ANSWERS!

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