An Overview of Other Topics, Conclusions, and Perspectives

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Machine Learning (CS771A)

Nov 11, 2016

Plan for today..

- Survey of some other topics
  - Sparse Modeling
  - Time-series modeling
  - Reinforcement Learning
  - Multitask/Transfer Learning
  - Active Learning
  - Bayesian Learning
- Conclusion and take-aways..

Sparse Modeling

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- Many ML problems can be written in the following general form

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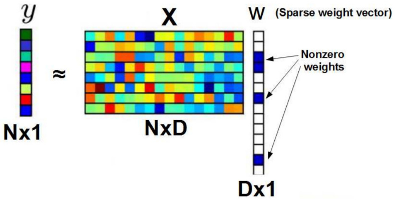

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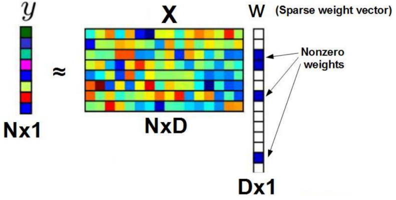

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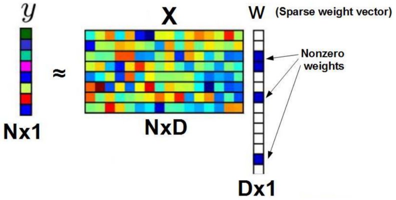

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- Examples: Sparse regression/classification, sparse matrix factorization, compressive sensing, dictionary learning, and many others.

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  - Use non-convex optimization methods, e.g., iterative hard threholding (rough idea: use gradient descent to solve for $\boldsymbol{w}$ and set $D - s$ smallest entries to zero in every iteration; basically a projected GD method)

† See "Optimization Methods for ℓ_1 Regularization" by Schmidt *et al* (2009)

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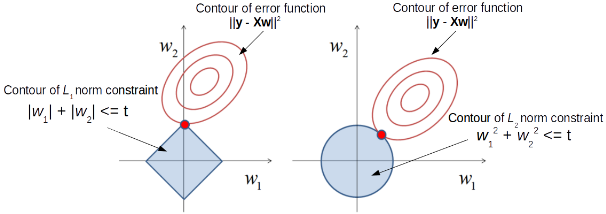

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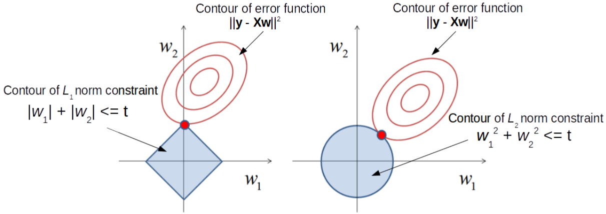
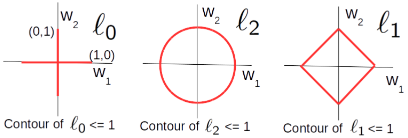


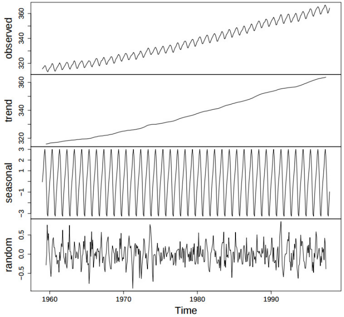
- Chances of the error function contour meeting the constraint contour at the coordinate axes is more likely in case of ℓ_1
- Another explanation: Between ℓ_2 and ℓ_1 norms, ℓ_1 is "closer" to the ℓ_0 norm (in fact, ℓ_1 norm is the closest convex approximation to ℓ_0 norm)



Learning from Time-Series Data

Modeling Time-Series Data

- The input is a sequence of (non-i.i.d.) examples $\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_T$
- The problem may be supervised or unsupervised, e.g.,
  - Forecasting: Predict $\boldsymbol{y}_{T+1}$, given $\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_T$
  - Cluster the examples or perform dimensionality reduction
- Evolution of time-series data can be attributed to several factors

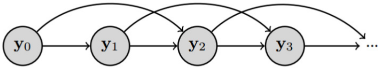


- Teasing apart these factors of variation is also an important problem

Auto-regressive Models

- Auto-regressive (AR): Regress each example on p previous examples

$$\boldsymbol{y}_t = c + \sum_{i=1}^{p} w_i \boldsymbol{y}_{t-i} + \epsilon_t \qquad : \text{ An AR}(p) \text{ model}$$



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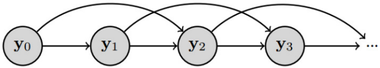

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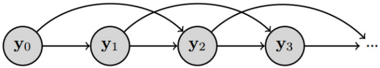

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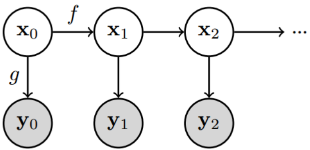
- Auto-regressive Moving Average (ARMA): Regress each example of p previous examples and q previous stochastic errors

$$\boldsymbol{y}_t = c + \epsilon_t + \sum_{i=1}^{p} w_i \boldsymbol{y}_{t-i} + \sum_{i=1}^{q} v_i \epsilon_{t-i} \qquad : \text{ An ARMA}(p, q) \text{ model}$$

State-Space Models

- Assume that each observation $\boldsymbol{y}_t$ in the time-series is generated by a low-dimensional latent factor $\boldsymbol{x}_t$ (one-hot or continuous)



State-Space Model (shown above: 1st order SSM)

- Basically, a generative latent factor model: $\boldsymbol{y}_t = g(\boldsymbol{x}_t)$ and $\boldsymbol{x}_t = f(\boldsymbol{x}_{t-1})$, where g and f are probability distributions
  - Very similar to PPCA/FA, except that latent factor $\boldsymbol{x}_t$ depends on $\boldsymbol{x}_{t-1}$
- Some popular SSMs: Hidden Markov Models (one-hot latent factor $\boldsymbol{x}_t$), Kalman Filters (real-valued latent factor $\boldsymbol{x}_t$)
- Note: Models like RNN/LSTM are also similar, except that these are not generative (but can be made generative)

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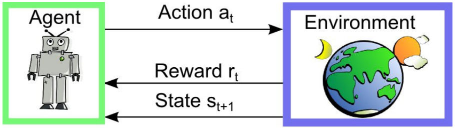

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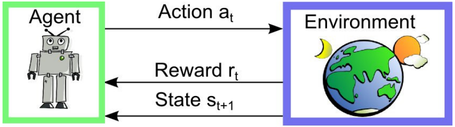

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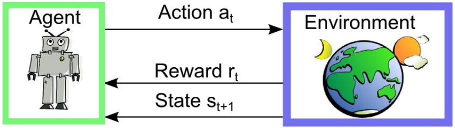

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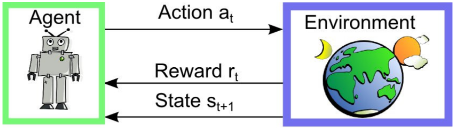

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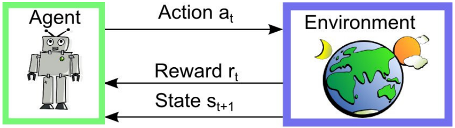

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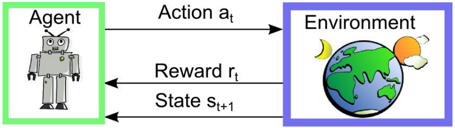

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- Agent's actions affect the subsequent data it receives
- Many applications: Robotics and control, computer game playing (e.g., Atari, GO), online advertising, financial trading, etc.

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  - (P_a, R) and π can be estimated in an alternating fashion
  - Estimating P_a and R requires some training data. Can be done even when the state space is continuous (requires solving a function approximation problem)

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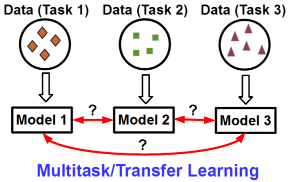

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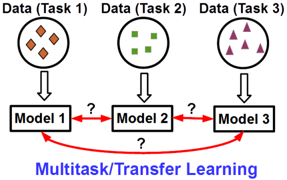

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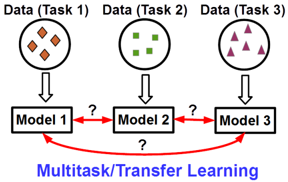

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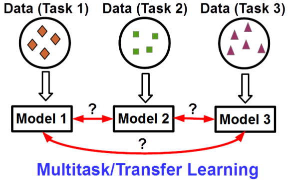

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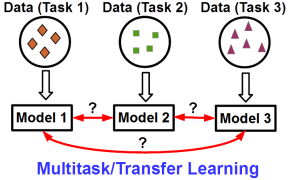

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  - Need to jointly regularize the models to make them similar to each other based on their degree/way of relatedness with each other (to be learned)

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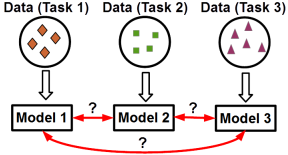

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  - Need to jointly regularize the models to make them similar to each other based on their degree/way of relatedness with each other (to be learned)
- Some other related problem settings: Domain Adaptation, Covariate Shift

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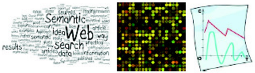
- Can actually estimate the ratio without estimating the densities (a huge body of work on this problem)

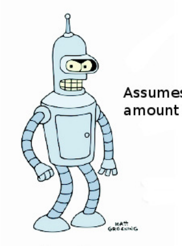
Active Learning

Active Learning

Allow the learner to ask for the most informative training examples



raw unlabeled data
$x_1, x_2, x_3, \ldots$



Assumes some small
amount of initial labeled training data

active learner
induces a classifier

expert / oracle
analyzes experiments
to determine labels

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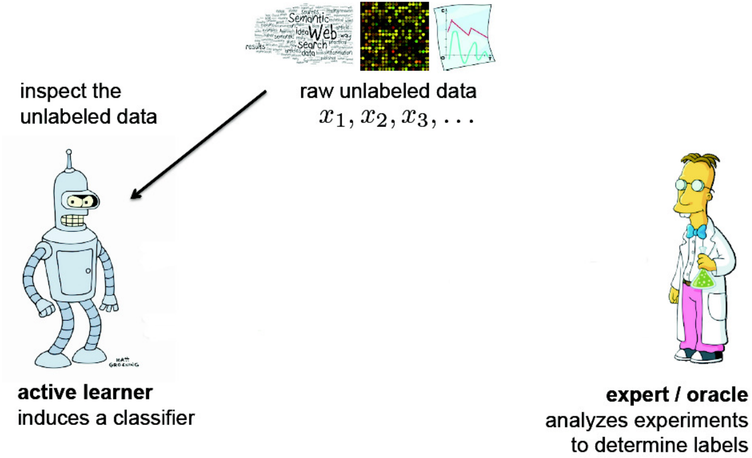

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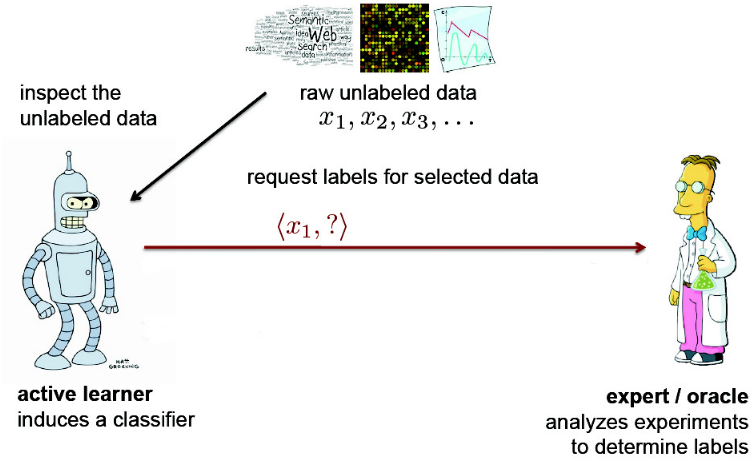

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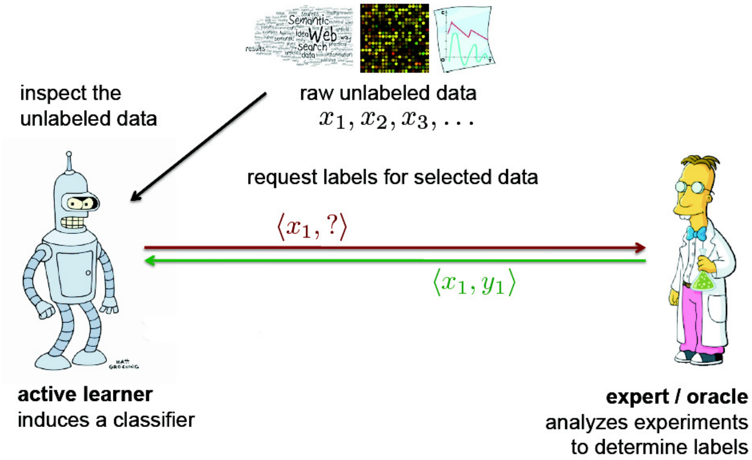

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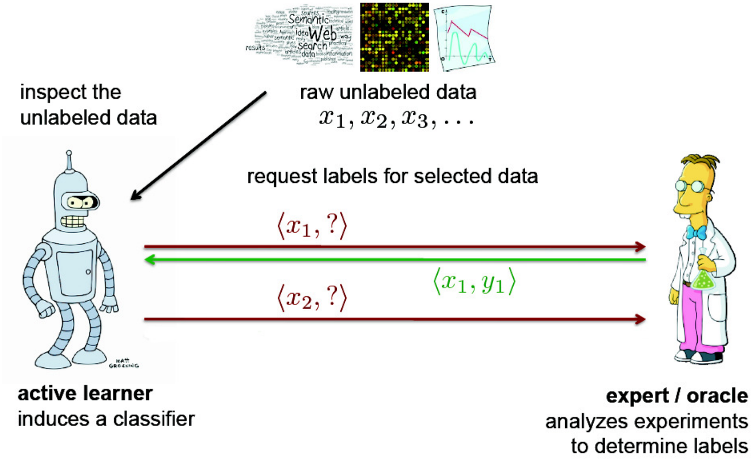

Bayesian Learning

Optimization vs Inference

Learning as Optimization

Learning as (Bayesian) Inference

Optimization vs Inference

Learning as Optimization

- Parameter θ is a fixed unknown

Learning as (Bayesian) Inference

Optimization vs Inference

Learning as Optimization

- Parameter θ is a fixed unknown
- Minimize a "loss" and find a point estimate (best answer) for θ, given data $\mathbf{X}$

$$\hat{\theta} = \arg\min_{\theta \in \Theta} \text{Loss}(\mathbf{X}; \theta)$$

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Learning as (Bayesian) Inference

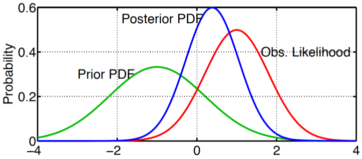
- Treat the parameter θ as a random variable with a prior distribution $p(\theta)$
- Infer a posterior distribution over the parameters using **Bayes rule**

$$p(\theta|\mathbf{X}) = \frac{p(\mathbf{X}|\theta)p(\theta)}{p(\mathbf{X})} \propto \text{Likelihood} \times \text{Prior}$$

- Posterior becomes the new prior for next batch of observed data
- No "fitting", so no overfitting!

Why be Bayesian?

- Can capture/quantify the uncertainty (or "variance") in θ via the posterior



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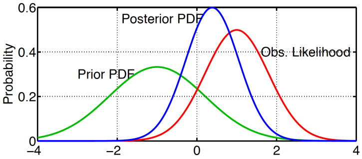

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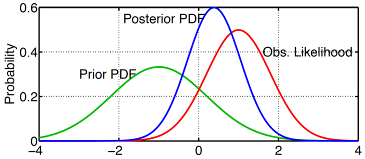

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- Many other benefits (wait for next semester :))

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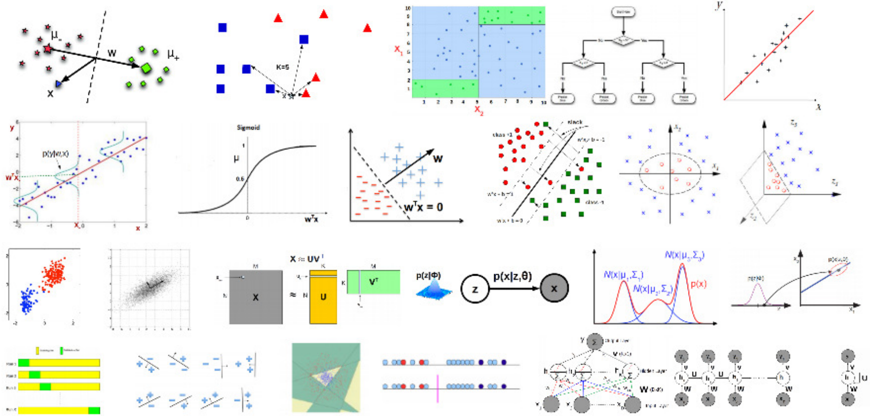
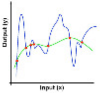
- Learn to first diagnose a learning algorithm rather than trying new ones
- No free lunch. No learning algorithm is "universally" good.

Thank You!

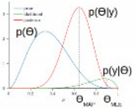


$$\hat{f} = \arg\min_f \sum_{n=1}^{N} \ell(y_n, f(\boldsymbol{x}_n)) + \lambda R(f)$$



$$p(\theta|\mathbf{y}) = \frac{p(\mathbf{y}|\theta)p(\theta)}{p(\mathbf{y})}$$

$$\hat{\theta}_{MAP} = \arg\max_\theta \sum_{n=1}^{N} \log p(y_n|\theta) + \log p(\theta)$$



And **No Free Lunch**!