Learning from Imbalanced Data

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Machine Learning (CS771A)

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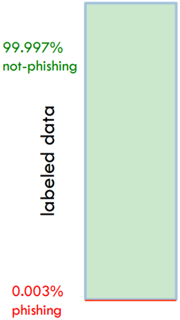
- Consider binary classification
- Often the classes are highly imbalanced



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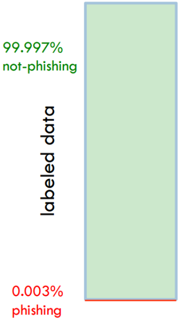
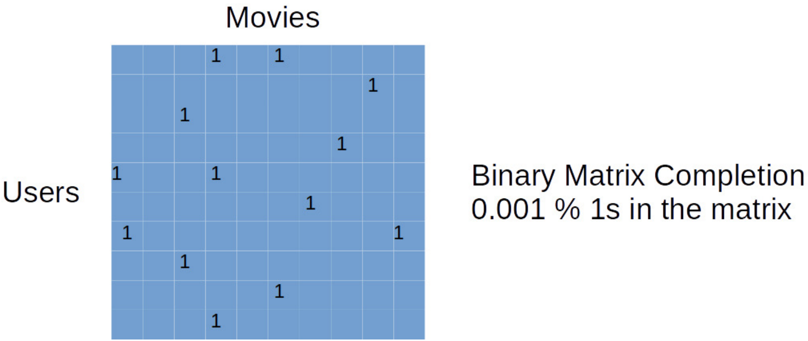


- Should I feel happy if my classifier gets 99.997% classification accuracy on test data ?

Learning from Imbalanced Data

- Other problems can also exhibit imbalance (e.g., binary matrix completion)



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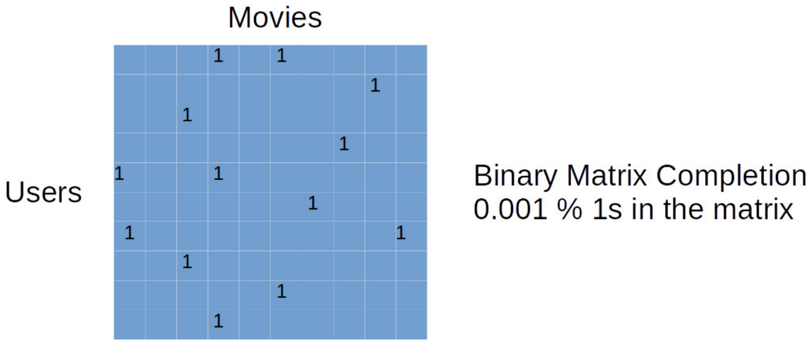

- Should I feel happy if my matrix completion model gets 99.999% matrix completion accuracy (or MAE close to 0) on the test entries?

True Definition of Imbalance Data?

- Debatable..
- Scenario 1: 100,000 negative and 1000 positive examples
- Scenario 2: 10,000 negative and 10 negative examples
- Scenario 3: 1000 negative and 1 negative example

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- Usually, imbalance is characterized by absolute rather than relative rarity
  - Finding needles in a haystack..

Minimizing Loss

- Any model to minimize the loss, e.g.,

$$\text{Classification:} \quad \hat{\boldsymbol{w}} = \arg\min_{\boldsymbol{w}} \sum_{n=1}^{N} \ell(y_n, \boldsymbol{w}^\top \boldsymbol{x}_n)$$

$$\text{Matrix Completion:} \quad (\hat{\mathbf{U}}, \hat{\mathbf{V}}) = \arg\min_{\mathbf{U},\mathbf{V}} ||\mathbf{X} - \mathbf{U}\mathbf{V}^\top||^2$$

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- However, it will be highly biased towards predicting the majority class
  - Thus accuracy alone can't be trusted as the evaluation measure if we care more about predicting minority class (say positive) correctly

Better Evaluation Measures

- Precision: What fraction of positive predictions is truly positive

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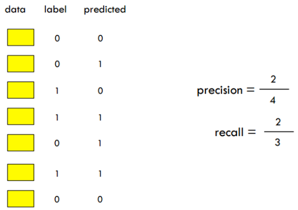

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data	label	predicted
	0	0
	0	1
	1	0
	1	1
	0	1
	1	1
	0	0

$$\text{precision} = \frac{2}{4}$$

$$\text{recall} = \frac{2}{3}$$

- Often there is a trade-off between precision and recall. Also these can be combined to yield other measures such as F1 score, AUC score, etc.

Dealing with Class Imbalance

- Modifying the training data (the class distribution)
  - Undersampling the majority class
  - Oversampling the minority class
  - Reweighting the examples
- Modifying the learning model
  - Use loss functions customized to handle class imbalance

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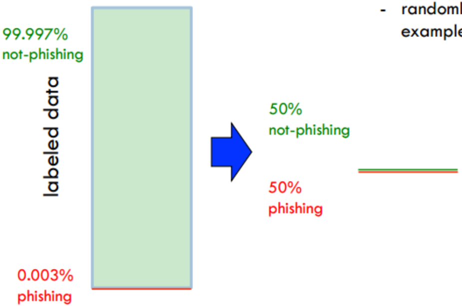
- Modifying the learning model
  - Use loss functions customized to handle class imbalance
- Reweighting can be also seen as a way to modify the loss function

Modifying the Training Data

Undersampling



Create a new training data set by:
- including all k “positive” examples
- randomly picking k “negative” examples

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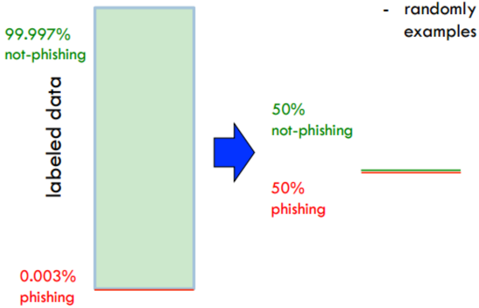


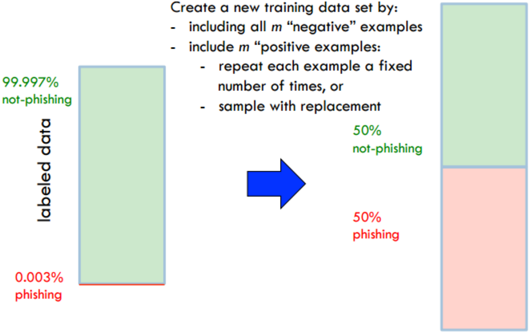
Create a new training data set by:
- including all k "positive" examples
- randomly picking k "negative" examples

- Throws away a lot of data/information. But efficient to train

Oversampling

Create a new training data set by:

- including all *m* “negative” examples
- include *m* “positive examples:
  - repeat each example a fixed number of times, or
  - sample with replacement



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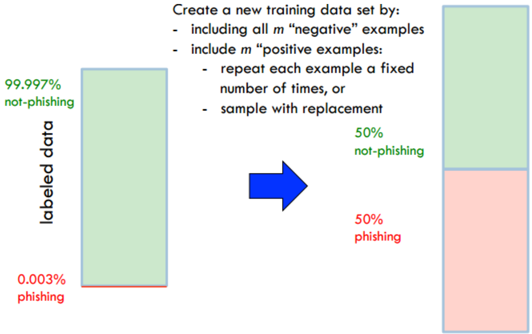

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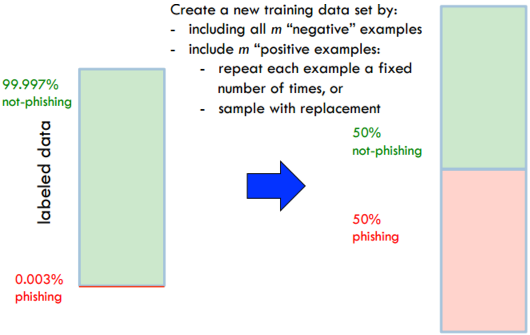

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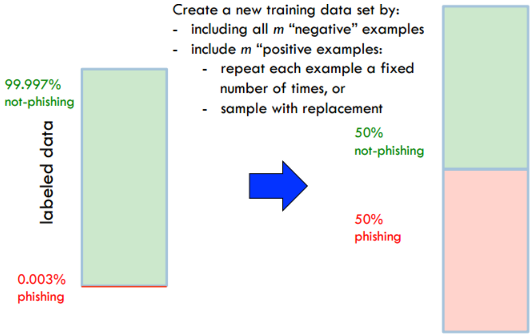

- From the loss function's perspective, the repeated examples simply contribute multiple times to the loss function
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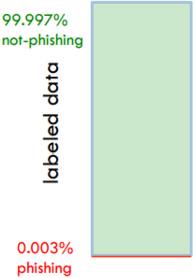
- Some oversampling methods (SMOTE) are based on creating synthetic examples from the minority class

Reweighting Examples



cost/weights

1

99.997/0.003 =
33332

Add costs/weights to the training set

"negative" examples get weight 1

"positive" examples get a much larger weight

change learning algorithm to optimize weighted training error

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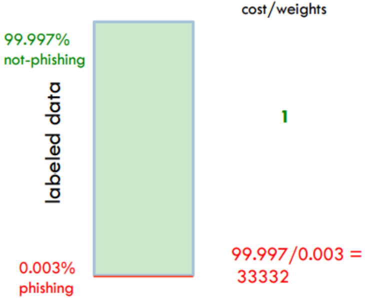

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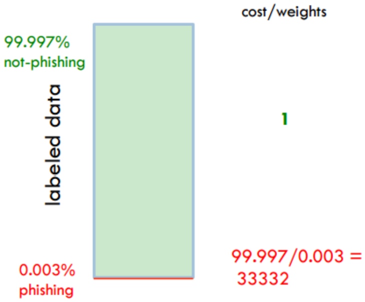

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- Similar effect as oversampling but is more efficient (because there is no multiplicity of examples)
- Also requires a classifier that can learn with weighted examples

Modifying the Loss Function

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- These are called "pairwise" loss functions
- Why is it a good loss function for imbalanced data?

Pairwise Loss Functions

- Using pairs with one +ve and one -ve doesn't let one class overwhelm other

$$\sum_{n=1}^{N_+}\sum_{m=1}^{N_-}\ell(f(\boldsymbol{x}_n^+), f(\boldsymbol{x}_m^-)) + \lambda R(f)$$

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- Note: Commonly used pairwise loss functions maximize a proxy of the AUC score (or closely related measures such as F1 score)

Pairwise Loss Functions

- A proxy based on hinge-loss like pairwise loss function for a linear model

$$\ell(\boldsymbol{w}, \boldsymbol{x}_n^+, \boldsymbol{x}_m^-) = \max\{0, 1 - (\boldsymbol{w}^\top \boldsymbol{x}_n^+ - \boldsymbol{w}^\top \boldsymbol{x}_m^-)\} = \max\{0, 1 - \boldsymbol{w}^\top (\boldsymbol{x}_n^+ - \boldsymbol{x}_m^-)\}$$

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- Note: Similar ideas can be used for solving binary matrix factorization and matrix completion problems as well
  - E.g., if matrix entry $X_{nm} = 1$ and $X_{nm'} = -1$ then loss=0 if $\boldsymbol{u}_n^\top \boldsymbol{v}_m > \boldsymbol{u}_n^\top \boldsymbol{v}_{m'}$

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- More principled approaches are based on modifying the loss function
  - Instead of minimizing the classication error, optimize w.r.t. other metrics such as precision, recall, F1 score, AUC, etc.

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- Sampling heuristics work reasonably on many data sets
- More principled approaches are based on modifying the loss function
  - Instead of minimizing the classication error, optimize w.r.t. other metrics such as precision, recall, F1 score, AUC, etc.
- Another way to look at this problem could be as an anomaly detection problem (minority class is anomaly) or density estimation problem