Deep Learning: Models for Sequence Data (RNN and LSTM)

Piyush Rai

Machine Learning (CS771A)

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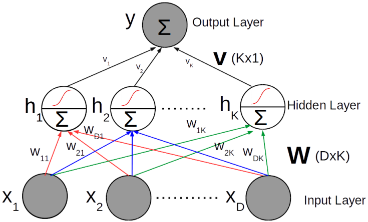
Recap: Feedforward Neural Network

- Consists of an input layer, one or more hidden layers, and an output layer



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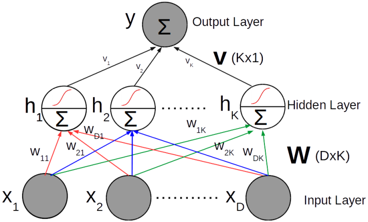
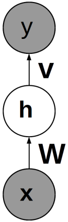


- A "macro" view of the above (note: $\boldsymbol{x} = [x_1, \ldots, x_D], \boldsymbol{h} = [h_1, \ldots, h_K]$)



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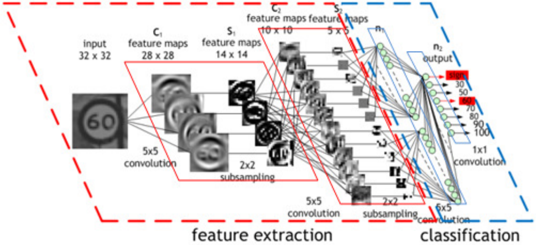

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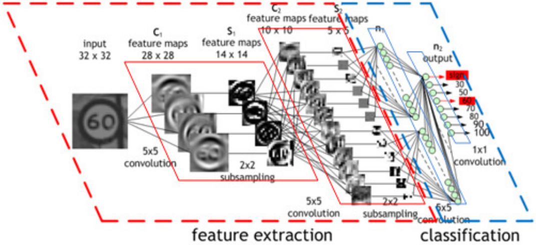

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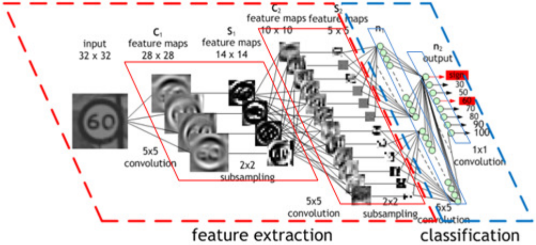

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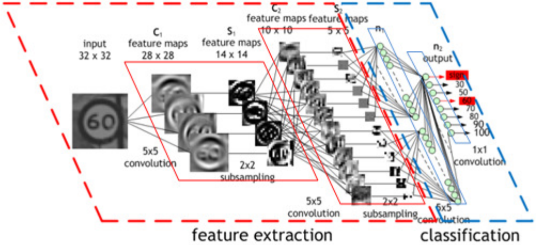

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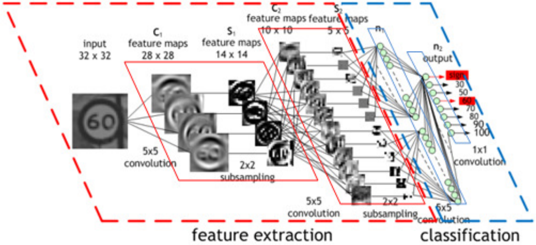

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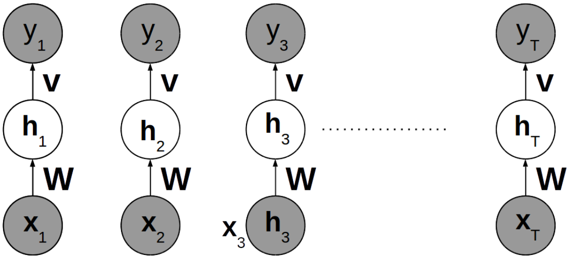
- Nonlinearity and pooling and applied after each convolution layer
- Last layer (one that connects to outputs) is fully connected

Deep Neural Networks for Modeling Sequence Data

Limitation of Feedforward Neural Nets

- FFNN can't take into account the sequential structure in the data
- For a sequence of observations $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_T$, their corresponding hidden units (states) $\boldsymbol{h}_1, \ldots, \boldsymbol{h}_T$ are assumed independent of each other



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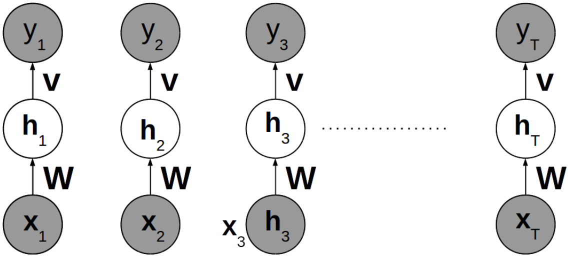
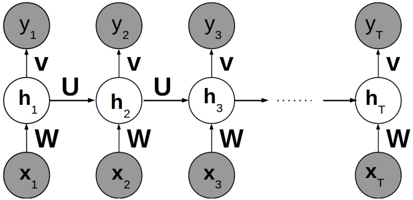


- Not idea for sequential data, e.g., sentence/paragraph/document (sequence of words), video (sequence of frames), etc.

Recurrent Neural Nets (RNN)

- Hidden state at each step depends on the hidden state of the previous



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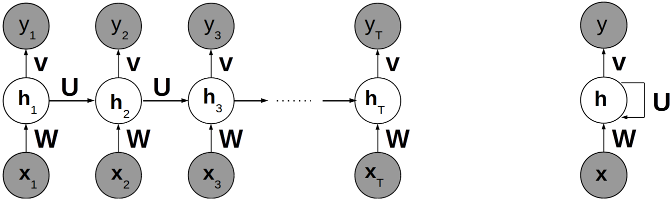

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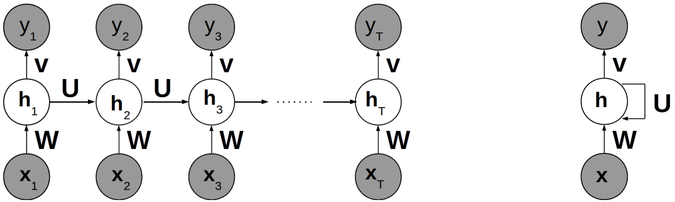

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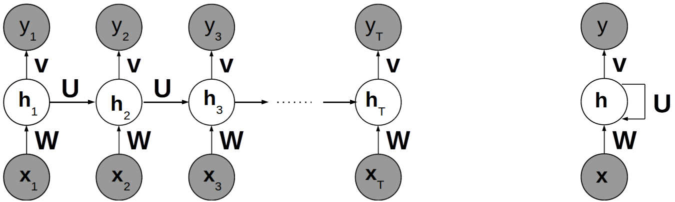

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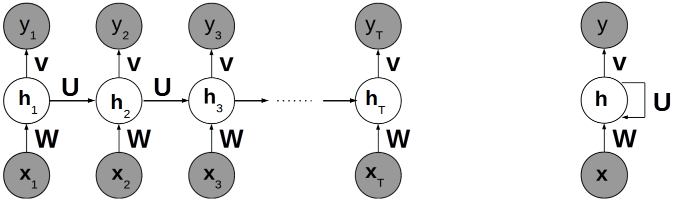

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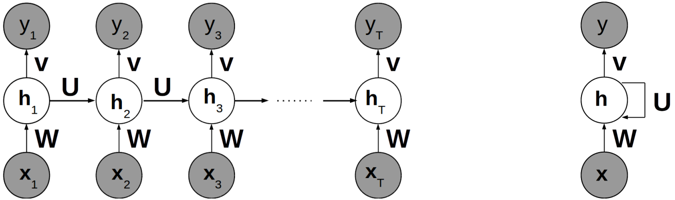

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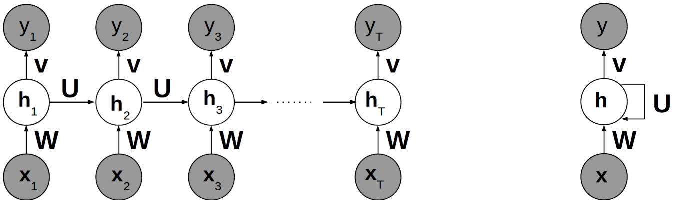

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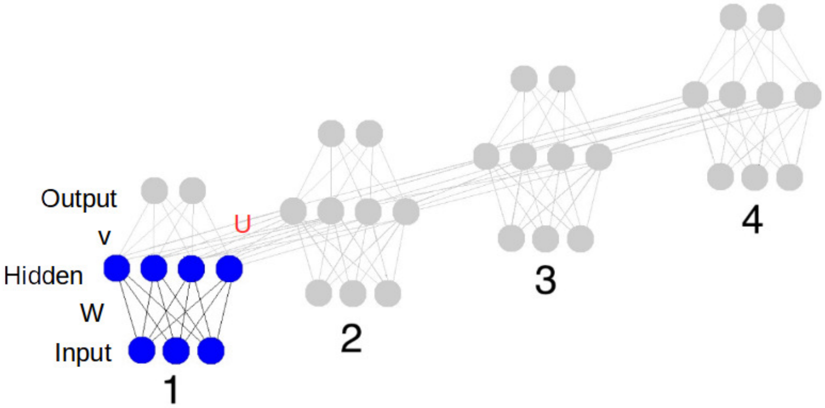
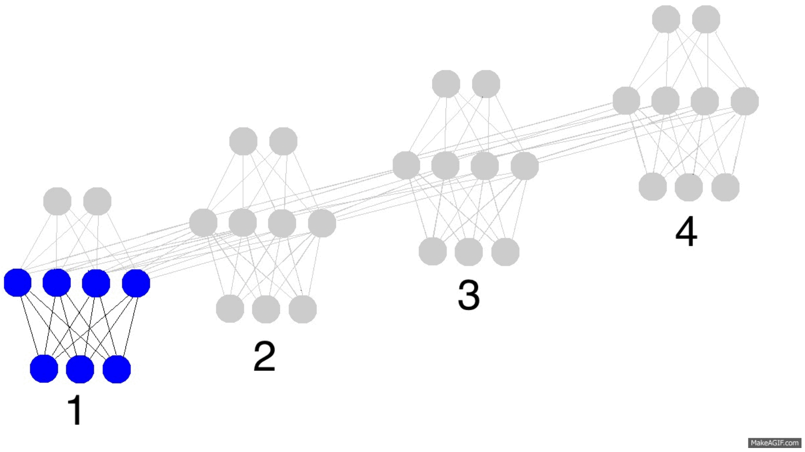
Recurrent Neural Nets (RNN)

- A more "micro" view of RNN (the transition matrix **U** connects the hidden states across observations, propagating information along the sequence)



RNN in Action..



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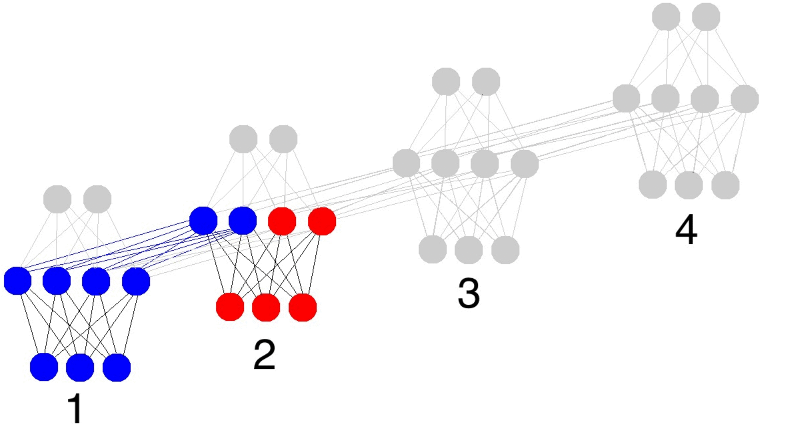

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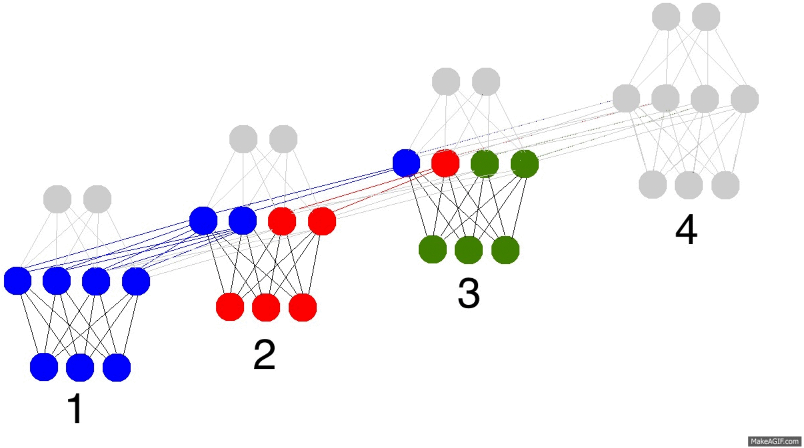

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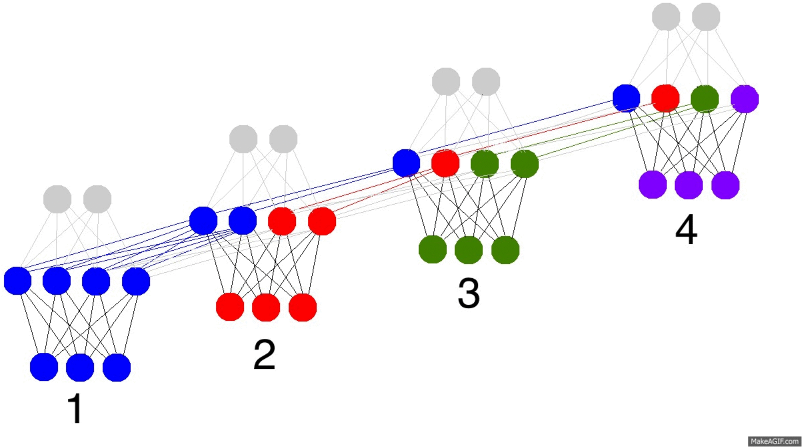
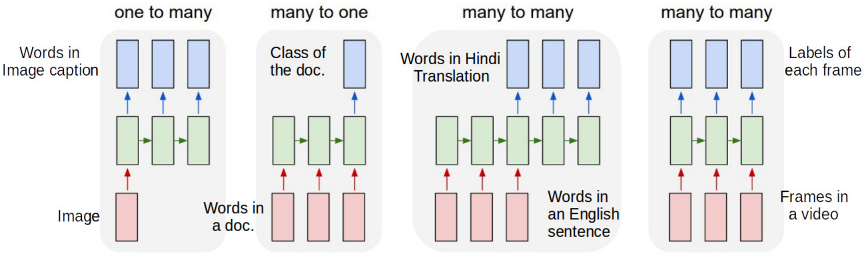


RNN: Applications



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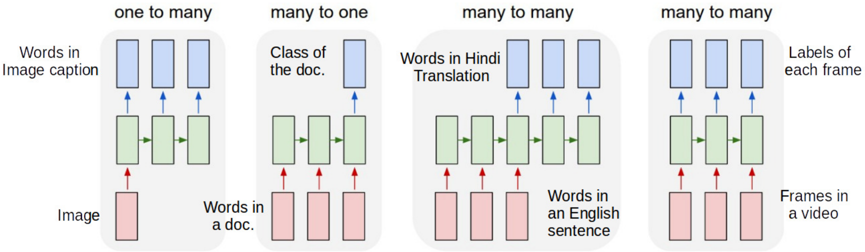

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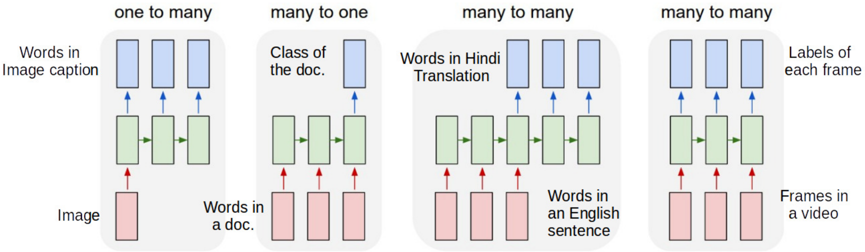

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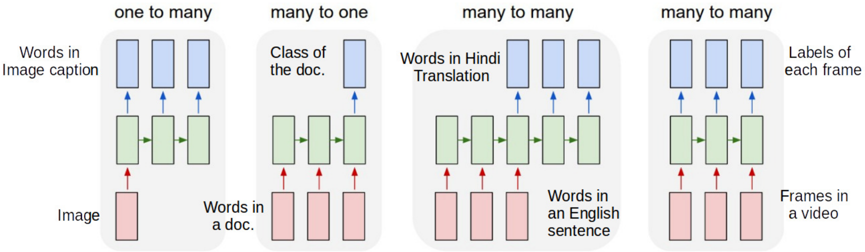

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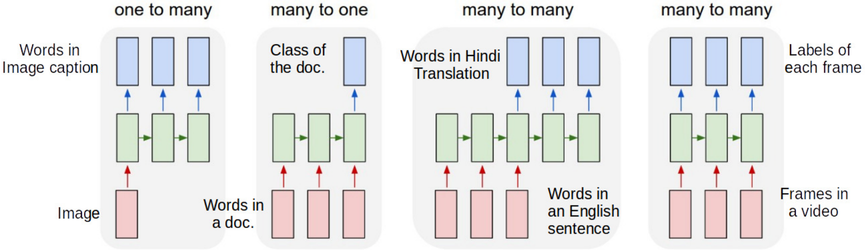

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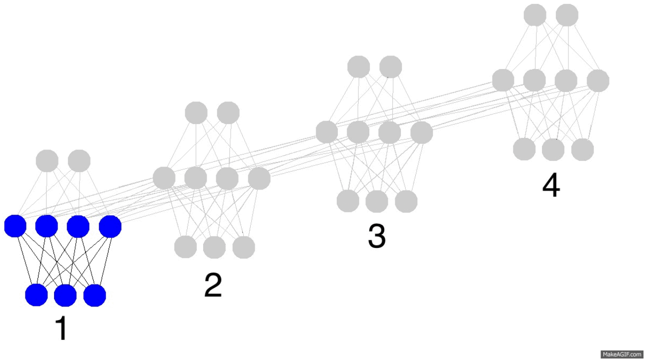
  - Input, output, or both, can be sequences (possibly of different lengths)
  - Different inputs (and different outputs) need not be of the same length
  - Regardless of the length of the input sequence, RNN will learn a fixed size embedding for the input sequence

Training RNN

- Trained using Backpropagation Through Time (forward propagate from step 1 to end, and then backward propagate from end to step 1)
- Think of the time-dimension as another hidden layer and then it is just like standard backpropagation for feedforward neural nets



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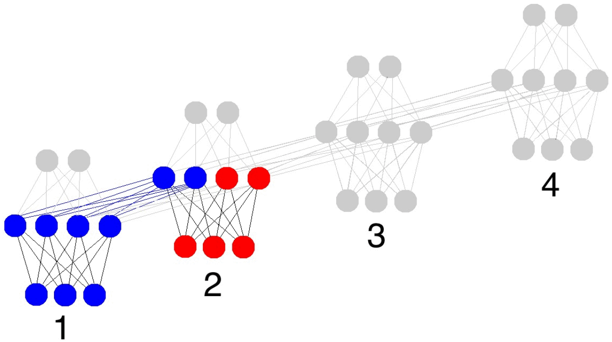

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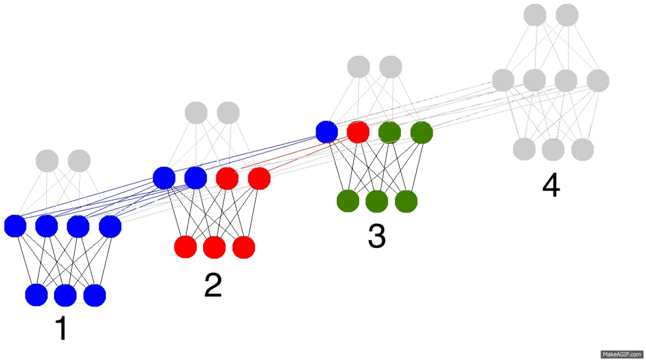

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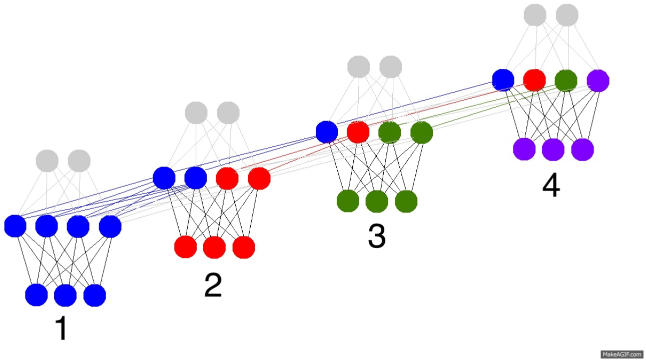

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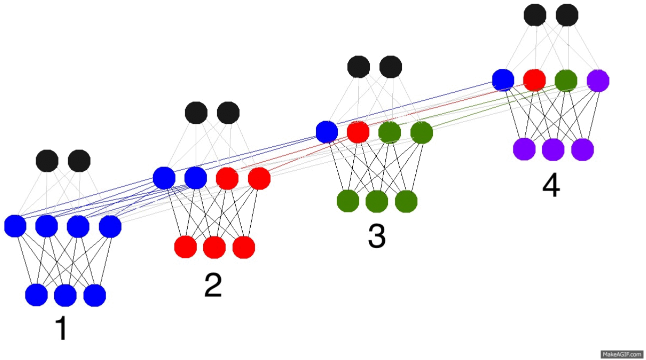

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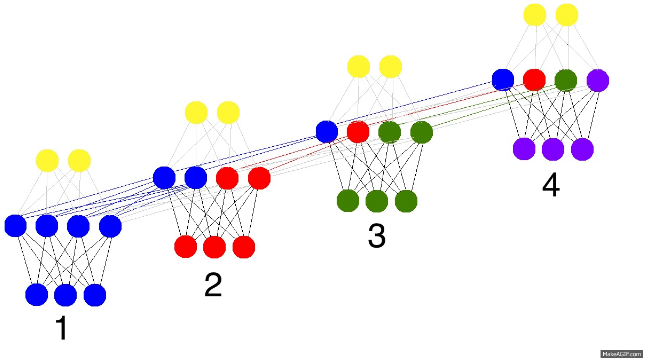

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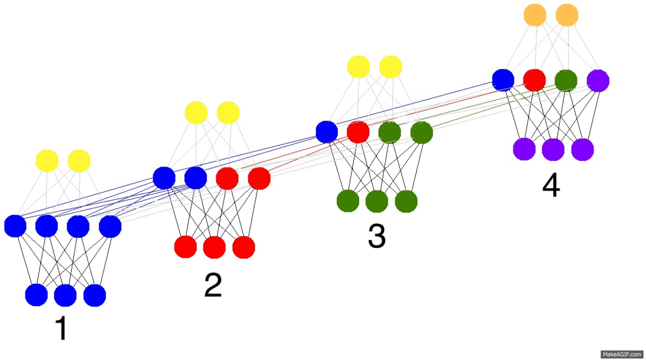

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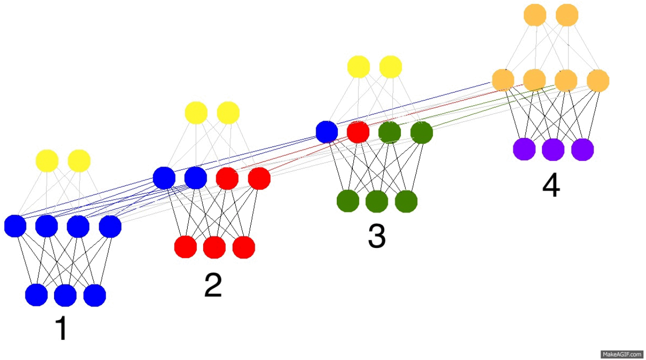

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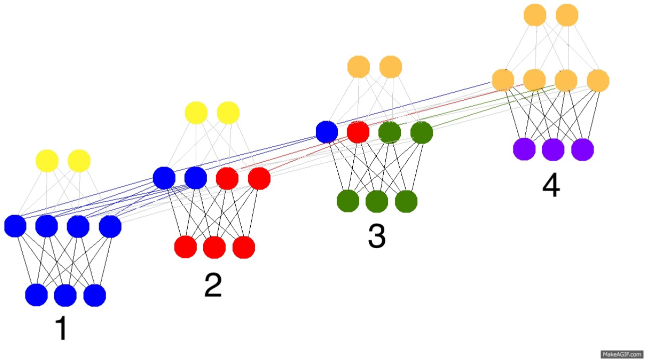

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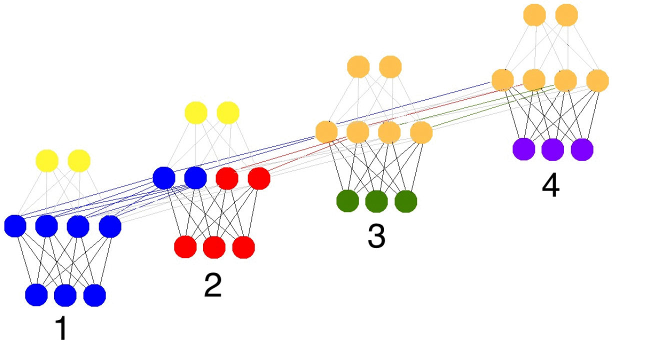

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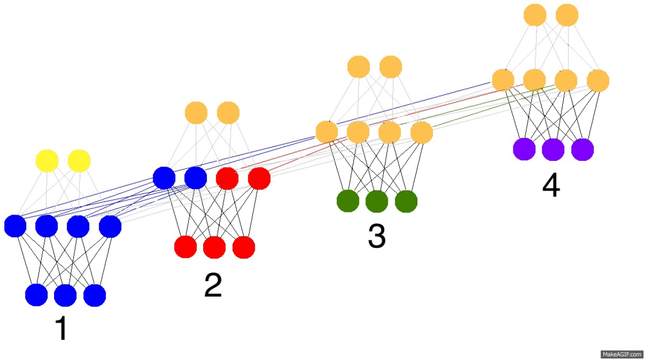

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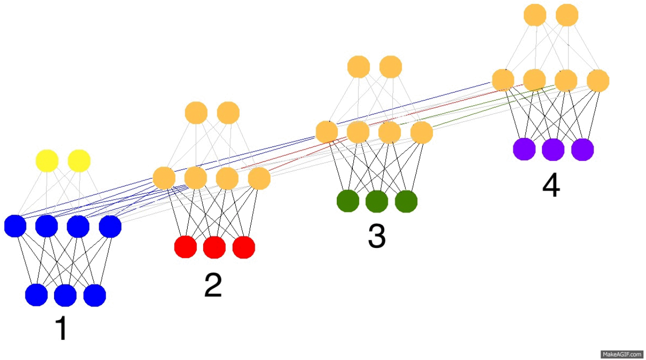

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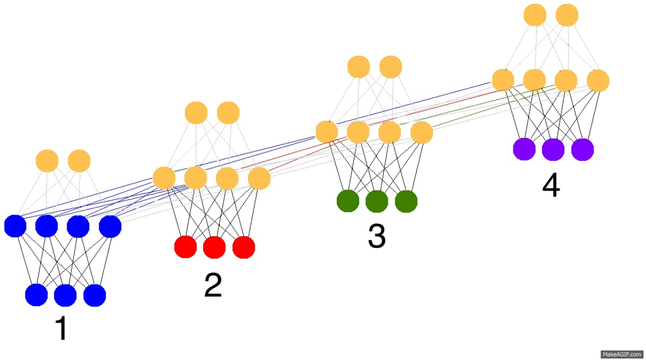

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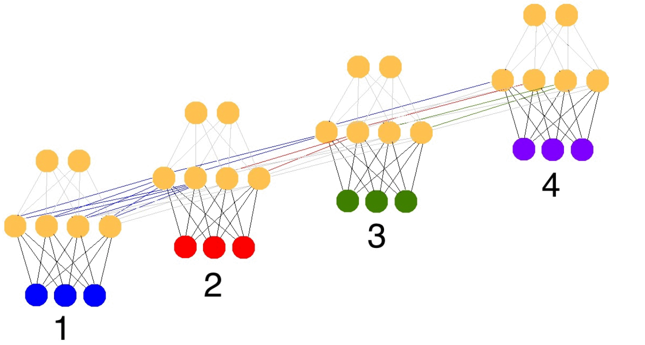

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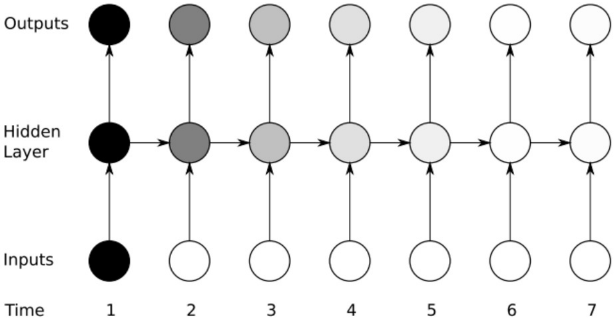
RNN: Vanishing/Exploding Gradients Problem



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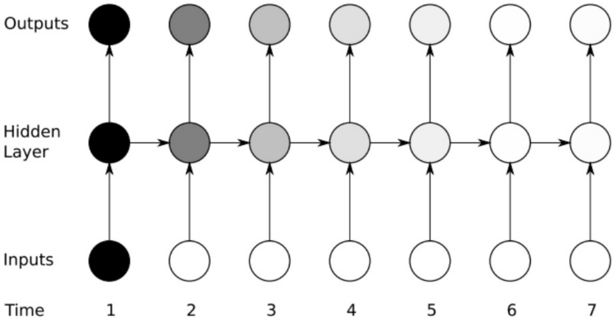

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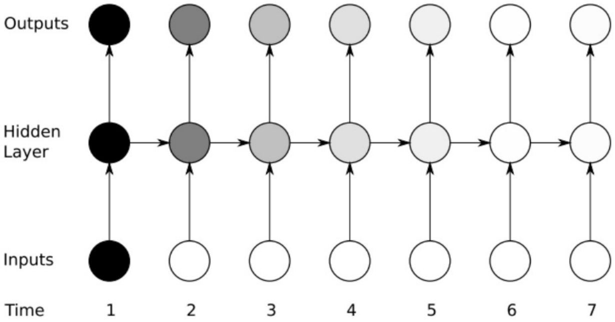

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- Repeated multiplications can cause the gradients to vanish or explode

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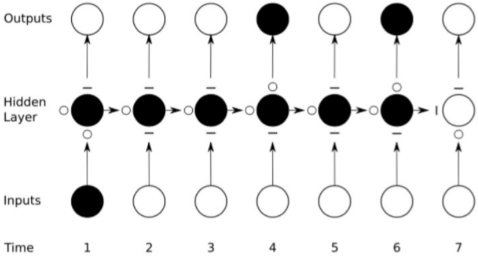
- Idea: Augment the hidden states with gates (with parameters to be learned)
- These gates can help us remember and forget information "selectively"



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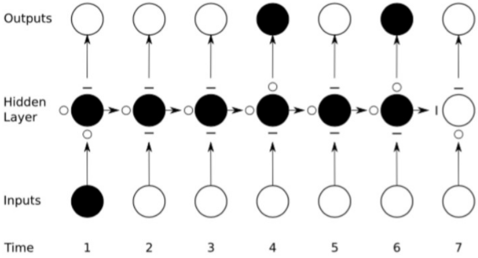

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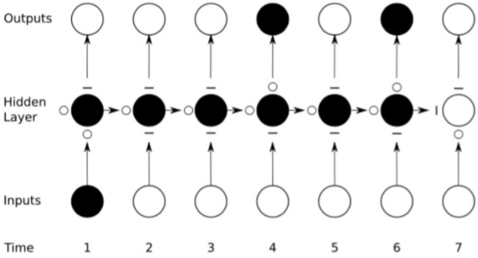

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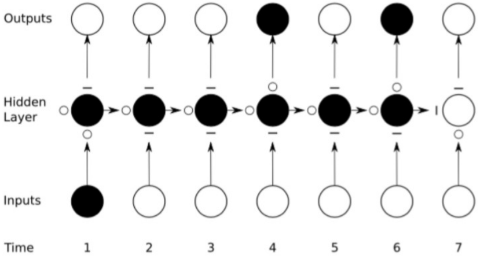
  - Input (bottom), Forget (left), Output (top)
- Open gate denoted by 'o', closed gate denoted by '-'

Capturing Long-Range Dependencies

- Idea: Augment the hidden states with gates (with parameters to be learned)
- These gates can help us remember and forget information "selectively"



- The hidden states have 3 type of gates
  - Input (bottom), Forget (left), Output (top)
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- LSTM (Hochreiter and Schmidhuber, mid-90s): **Long Short-Term Memory** is one such idea

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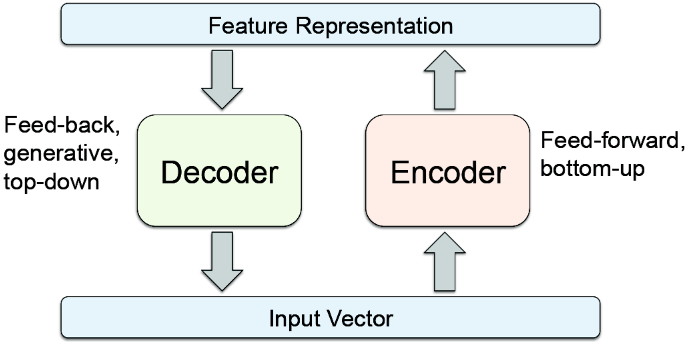
- Note: $\odot$ represents elementwise vector product. Also, state updates now additive, not multiplicative. Training using backpropagation through time.
- Many variants of LSTM exists, e.g., using C_{t-1} in local computations, Gated Recurrent Units (GRU), etc. Mostly minor variations of basic LSTM above

Neural Nets for Unsupervised Learning

Autoencoder

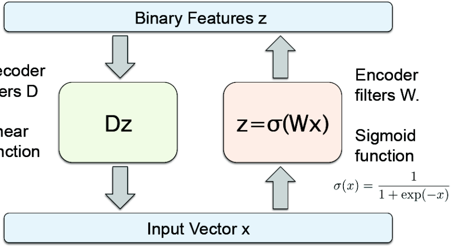
- A neural net for unsupervised feature extraction
- Basic principle: Learns an encoding of the inputs so as to recover the original input from the encodings as well as possible



- Also used to initialize deep learning models (layer-by-layer pre-training)

Autoencoder: An Example

- Real-valued inputs, binary-valued encodings



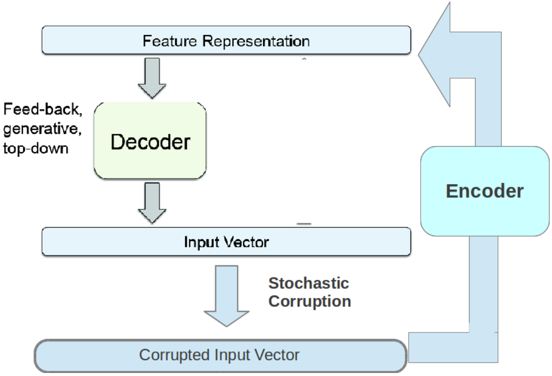
- Sigmoid encoder (parameter matrix W), linear decoder (parameter matrix D), learned via:

$$\arg\min_{D,W} E(D,W) = \sum_{n=1}^{N} ||D\mathbf{z}_n - \mathbf{x}_n||^2 = \sum_{n=1}^{N} ||D\sigma(W\mathbf{x}_n) - \mathbf{x}_n||^2$$

- If encoder is also linear, then autoencoder is equivalent to PCA

Denoising Autoencoders

- Idea: introduce stochastic corruption to the input; e.g.:
  - Hide some features
  - Add gaussian noise



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    - Modeling and retaining context is important when modeling sequential data (desirable to have a "memory module" of some sort as in LSTMs)
- Looked at Autoencoder - Neural network for unsupervised feature extraction
- Didn't discuss some other popular methods, e.g., deep generative models, but these are based on similar underlying principles