LQG-MP: Optimized path planning for robots with motion uncertainty and imperfect state information

Jur van den Berg, Pieter Abbeel and Ken Goldberg
Review by Harshad Sawhney

Abstract

This paper deals with improvements in robot motion panning that takes into account the uncertainty in sensors and control inputs. The path planning methods may provide an optimal path but they do not consider the errors that are caused due to external physical factors like uncertainty in environment, motors, sensors and incomplete information due to partial sensing which deviates the robot from the optimal path. Thus, the robot may not travers the optimal path and the errors could even lead to non feasible paths. The above errors suggest the need to integrate the path planning process with the input of the sensors and controllers and hence taking the physical factors into account.

Previous Work

- Markov Decision process or partially observable Markov decision process were proposed for sensing uncertainty. (Porta et al. 2006)
- LaVale and Hutchinson use a global control policy in case of motion and sensing uncertainty.
- There exists planners that take into account the sensing capability. The method proposed by Pepy and Lambert used extended Kalman filter to remove the uncertainty in the position of the robot. They used RRT algorithm to plan the path. Their method drawbacks included the time needed for finding a safe path.
- Prentice and Roy also use Kalman filter style estimators to predict the paths and presented a variant of the probabilistic roadmap technique called the belief roadmap.

The two above methods that are most closely related to this method use the maximum likelihood observations along the path rather than using the true probability distributions of the state of the robot and both the above papers did not take into account the controller uncertainty during execution of the path.

Claims

A set of 1000 candidate paths are evaluated using path planning methods. The method used in the paper is Rapidly exploring Random Trees(RRT). Using the dynamics and observation model, the linear quadratic controller with Gaussian model of uncertainty calculates the a priori distribution of state and control of the robot along all the candidate paths. Thus, it optimizes the planned path for the robot. The method is also applied to pre-computed roadmaps to find optimal paths.

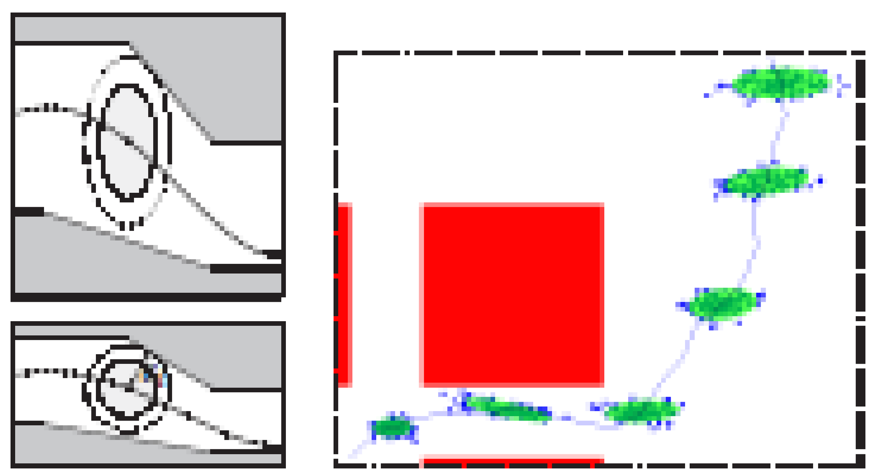

Fig1: (a) The maximum factor c_t by which the ellipse can be scaled not intersecting with obstacles.
(b) Ellipses show a priori distribution computed by LQG-MP.

(a) (b)

Method

$\chi = \mathbb{R}^n$ – state space $\quad U = \mathbb{R}^m$ – control input space

Standard formulation of model for LQG-control

- Stochastic Dynamics Model with motion uncertainty:

$x_t = f(x_{t-1}, u_{t-1}, m_t),\ m_t \sim N(0, M_t)$

- Stochastic Observation Model with sensor measurement noise:

$z_t = h(x_t, n_t),\ n_t \sim N(0, N_t)$

a) Kalman filter provides the optimal state estimate using previous state estimate, measurements and control inputs.
   1. Process update- It propagates the applied control input $\bar{u}_t$
   2. Measurement update- It obtains sensor measurement $\bar{z}_t$

b) LQR controller provides optimal control input using state estimate. $\quad \bar{u}_t = L_{t+1}\, \tilde{x}_t$

where L_{t+1} is the feedback matrix and $\tilde{x}_t$ is the estimated state. Kalman gain matrices are calculated through forward recursion while LQR feedback matrices are calculated through backward recursion in advance.

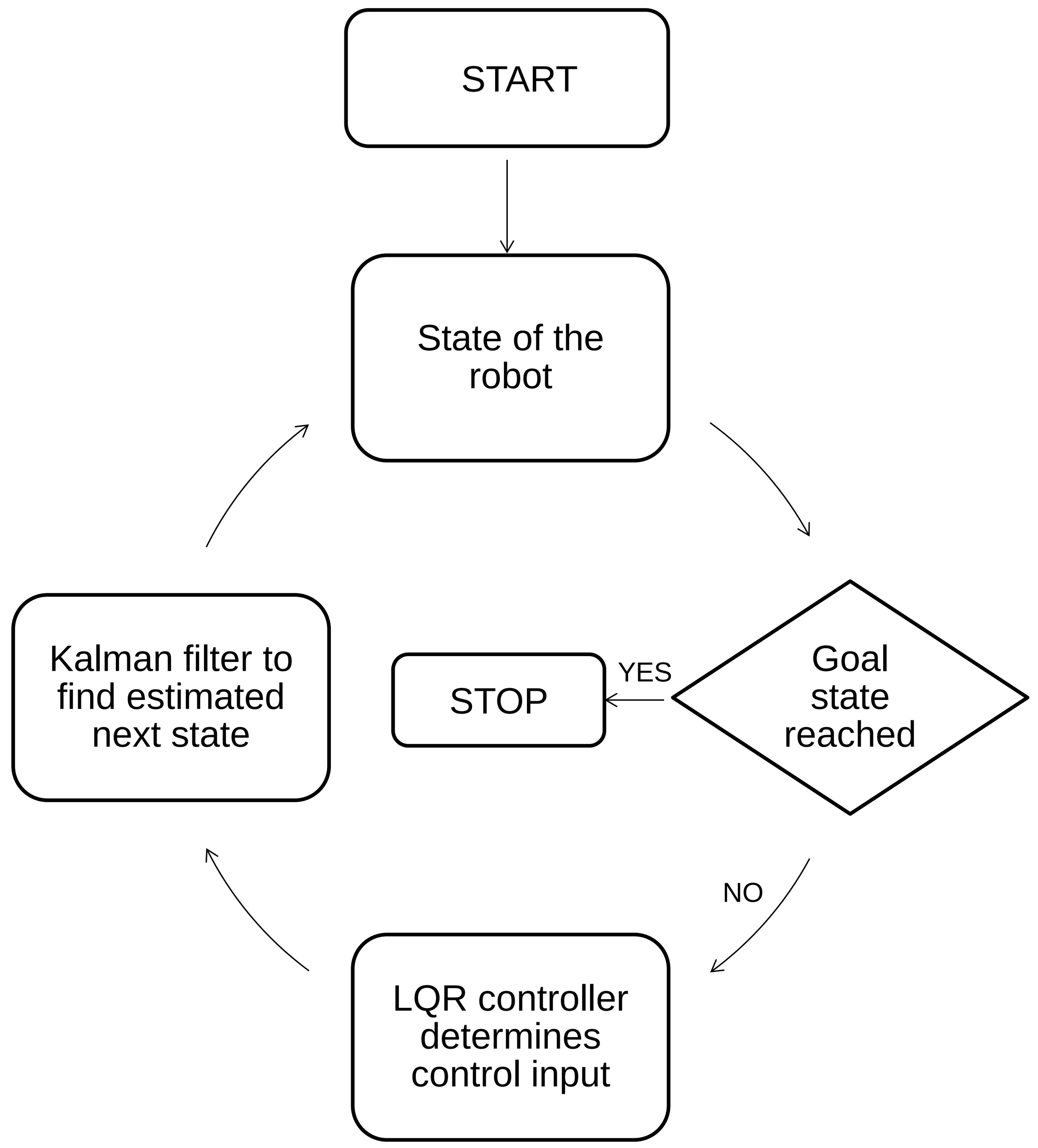

Fig2: Flow chart depicting the approach to find optimum path

Car Like Robot

State: (x, y, ϴ, v) $\quad$ Control Input: (α, φ)

Partial sensing- robot receives feedback only in y-direction

Controller noise: $m = (\tilde{\alpha}, \tilde{\varphi})$ $\quad$ Sensing noise: $n = \tilde{y} \sim N(0, \sigma_y^2)$

We need to minimize the probability of collision with obstacles. For this, we find the number of standard deviations that the robot can deviate before collision with obstacles which is denoted by c_t as shown in Fig1. Quality of path = $\prod_{t=0}^{l} \Gamma(\frac{n}{2}, \frac{c_t^2}{2})$

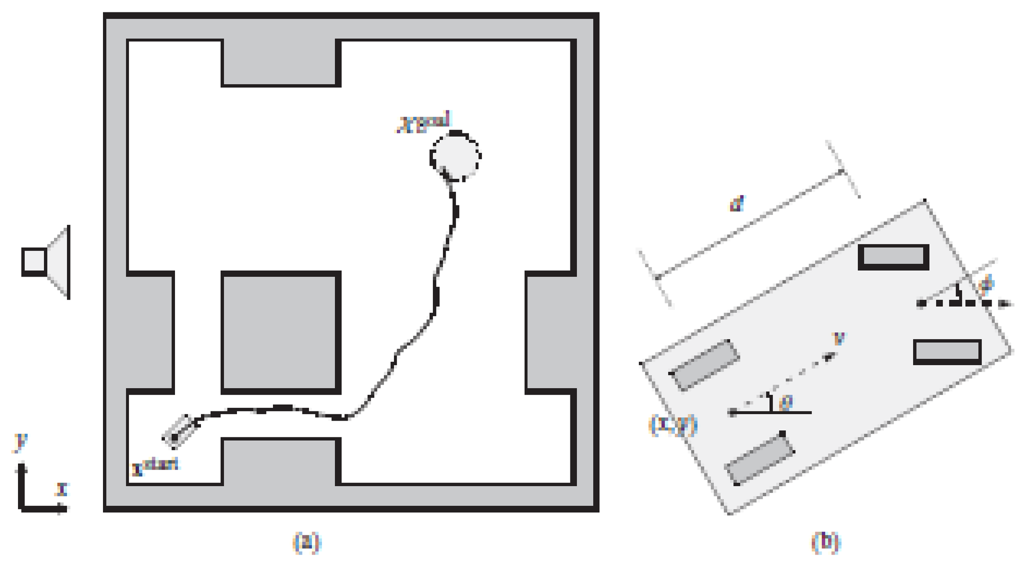

Fig3: Robot has to move from start to goal position with sensing in only y-coordinate

Results: Best path has 99% success rate using LQG-MP in low noise level environment.

Multi robots

8 robots with differential drive motion were used as shown in Fig4 which had to travel simultaneously avoiding mutual collisions.

State **x** = (x, y, ϴ) $\quad$ Control Input **u** = (v_l, v_r)

Process noise **m** = $(\tilde{v}_l, \tilde{v}_r) \sim N(0, \sigma_v^2 I)$

5 beacons signals are used to provide sensing signal that decays quadratically with distance to beacon.

n = $(\tilde{b}_1, \ldots, \tilde{b}_5) \sim N(0, \sigma_b^2 I)$.

The aim was to minimize probability of collisions.

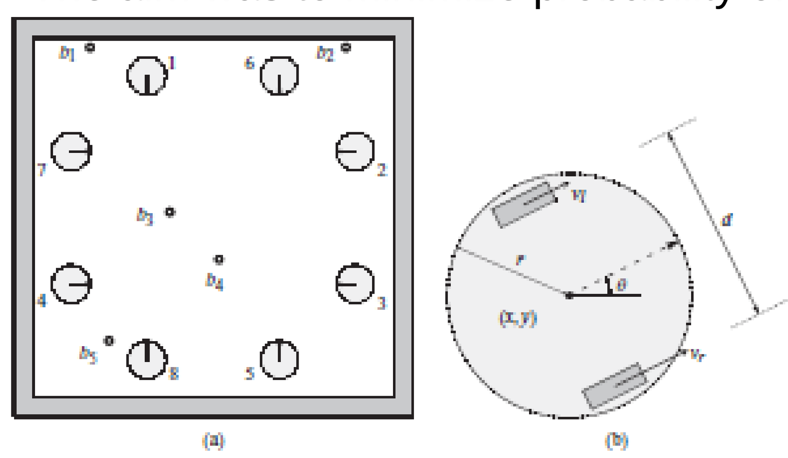

Fig4: Priorities were assigned to each robot for path planning, thus higher priority robots acted as moving obstacles for lower ones.

Results: Best path has 98.6% success rate using LQG-MP as compared to 2.13% for using the path obtained through RRT.

6-DOF manipulator

The aim is to maximize the likelihood of the end effector to reach goal position which is done by minimizing the variance of end-effector's position at the last stage of path.

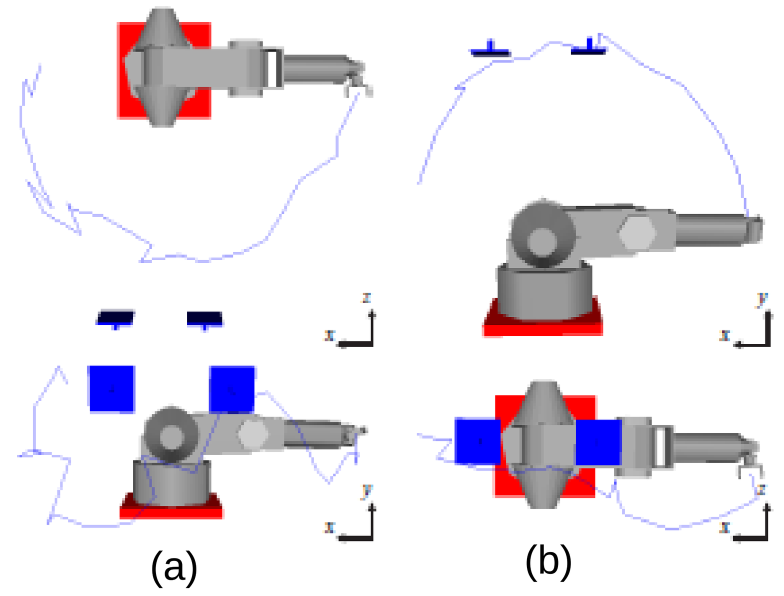

Fig5: The top and side views are shown. (a) The cameras are placed beside the robot (b) The cameras are placed above the robot

State **x** = $(\Theta_1, \ldots, \Theta_6)$ $\quad$ Control Input **u** = $(\omega_1, \ldots, \omega_6)$

Robot receives feedback from stereo camera.

m = $(\tilde{\omega}_1, \ldots, \tilde{\omega}_6) \sim N(0, \sigma_\omega^2 I)$ $\quad$ **n** $\sim N(0, \sigma_n^2 I)$

Results: The robot moves in the plane parallel to the viewing direction of the camera. It also stretches the end-effector fully. To bring the end-effector closer to camera for precise position.

Kalman smoothing

Randomized path planning algorithms generate non-smooth paths which may lead to collisions with obstacles. A pseudo dynamics model is assumed and the constraint on the magnitude of k_{th} order difference of control input **u** smoothens the path.

Roadmap

The motivation behind using roadmaps is to reduce the computation time significantly. A variant of Dijkstra's algorithm is used to find a path which uses LQG-MP. The probability that the path is collision free is the product of probabilities along each edge which is maximized.

Assumptions and Limitations

- Linear dynamics and observation model is assumed. Non-linear models with local linearization is used. The linearization is valid for small noise factors and accurate measurements up to that can be practically achieved.
- The noise is assumed to be Gaussian in nature. The results obtained in the simulation justify the assumption.
- There should be no occlusion in the field of view and sensors should not be conditional in nature.
- The experiment has only used the a priori distribution of sensors, the a priori distribution of control inputs is not used.
- The planning times reported in the experiment do not permit real time applications.

Conclusion

The simulations achieve accurate results. Thus, the method of computing a priori probability distribution of the state of the robot to optimize a task precisely achieves its purpose.

References

[1] Prentice, Samuel, and Nicholas Roy. "The belief roadmap: Efficient planning in belief space by factoring the covariance." *The International Journal of Robotics Research* 28.11-12 (2009).

[2] LaValle, Steven M., and Seth A. Hutchinson. "An objective-based framework for motion planning under sensing and control uncertainties."*The International Journal of Robotics Research* 17.1 (1998): 19-42.

[3] Pepy, Romain, and Alain Lambert. "Safe path planning in an uncertain-configuration space using RRT." *Intelligent Robots and Systems, 2006 IEEE/RSJ International Conference on*. IEEE, 2006.